Comparative Study of Vertical Motions in the Atmosphere Evaluated by Various Schemes

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1. INTRODUCTION

This study compares vertical motions in the global atmosphere, evaluated by various schemes of integrating the continuity equation. The schemes include a central difference scheme, plane fitting method, spherical harmonic expansion method, and normal mode expansion method.

For comparison, the vertical motions provided by NCEP and ECMWF reanalyses and ECMWF TOGA analysis are presented for the reference.

2. DATA AND METHOD

The twice daily (0000 and 1200 GMT) NCEP global reanalysis with u, v, ω, T, ϕ are defined at every 2.5 by 2.5 degree grid points over 17 vertical levels from 1000 to 10 hPa. Since ω is provided in the NCEP and ECMWF reanalyses, the results in this study are compared with these ω to find to what extent these values agree with each other.

The vertical motions ω are evaluated from horizontal divergence ($\nabla \cdot \mathbf{V}$) in pressure (p) coordinate by integrating the continuity equation in the vertical.

$$\nabla \cdot \mathbf{V} + \frac{\partial \omega}{\partial p} = 0 \quad \rightarrow \quad \omega = -\int_0^p \nabla \cdot \mathbf{V}\, dp. \quad (1)$$

Here, the boundary condition should be ω=0 at the limit of $p \rightarrow 0$.

2.1 Difference Scheme

In this scheme, the horizontal divergence is approximated by a standard central finite difference method:

$$\begin{aligned} \nabla \cdot \mathbf{V} &= \frac{u(x+\Delta x) - u(x-\Delta x)}{2\Delta x} \\ &+ \frac{v(y+\Delta y) - v(y-\Delta y)}{2\Delta y} \end{aligned} \qquad (2)$$

The divergence is evaluated by the small difference in wind speed. Unfortunately, this scheme contains large error in small scales since the observed wind has, at least, 10 percent of error. Hence, the divergence sometimes contains more than 100 percent of error.

2.2 Plane Fitting Method

This method is aimed to remove such an erroneous divergence in small scales by fitting wind field on a plane by the lease square method using adjacent data around the origin. The deviations from the plane are considered mostly as an observational error.

$$\begin{aligned} u(x,y) &= u_0 + ax + by \\ v(x,y) &= v_0 + cx + dy \\ \nabla \cdot \mathbf{V} &= a + d \end{aligned} \qquad (3)$$

This method is widely used since it can be applicable for randomly distributed observation stations.

2.3 Spectral Method

Since the fundamental idea of the successful Plane Fitting Method is a filtering of divergence in small scales, it is straightforward to apply the method for global data by a spectral expansion and synthesis.

$$\nabla \cdot \mathbf{V} = \delta = \sum_m \sum_n \delta_n^m P_n^m(\mu) e^{im\lambda}$$

$$\begin{aligned}\delta_n^m &= \frac{1}{\|P_n^m\|^2}\int_{-1}^{1}\frac{imU^m}{a(1-\mu^2)}P_n^m \\ &- \frac{V^m}{a(1-\mu^2)}H_n^m d\mu \qquad (4)\end{aligned}$$

Here, the divergence is expanded in spherical harmonics and synthesized over the wavenumbers approximately a half of the Nyquist wavenumber. The expansion coefficients are evaluated analytically from the observed wind without a use of finite difference scheme.

Although the divergence should be integrated from the top to bottom of the atmosphere with the boundary condition of $\omega=0$ at the limit of $p \to 0$, it has been commonly integrated from the bottom to top for abovementioned schemes by evaluating surface ω under a proper assumptions. The resulting contradiction at the top boundary are adjusted by O'Brine's (1970) quadratic correction. However, the surface ω appears to be erroneous over the mountainous regions, such as Tibettern Plateau and Antarctic.

2.4 Normal Mode Method

In this study, we have developed a new spectral method to calculate ω by expanding the variable in 3-D normal mode functions:

$$\begin{aligned}\mathbf{U}(\lambda,\theta,p,t) &= \sum_{nlm} w_{nlm}(t)\mathbf{X}_m\mathbf{\Pi}_{nlm}(\lambda,\theta,p), \\ w_{nlm}(t) &= < \mathbf{U},\mathbf{X}_m^{-1}\mathbf{\Pi}_{nlm} > . \qquad (5)\end{aligned}$$

Here,

$$\mathbf{U} = (u, v, \phi)^T \qquad (6)$$

is the state variable vector with horizontal wind velocity u, v and geopotential deviation from the reference state ϕ as functions of longitude λ, latitude θ, pressure p and time t. The 3-D Fourier expansion coefficient w_{nlm} has triple subscripts of zonal, meridional and vertical wavenumbers, respectively. The expansion basis function Π_{nlm} is a tensor product of vertical structure functions G_m and Hough functions. The dimensional factor matrix X_m contains proper scale parameters involving the separation constant of the equivalent depth h_m. The equations in (5) construct a pair of Fourier transforms in the 3-D spectral domain.

Once the expansion coefficient w_{nlm} is obtained, the vertical motion ω may be calculated by the synthesis, corresponding to the inverse transform:

$$\omega = \sum_{nlm} 2\Omega\frac{gh_m p^2}{R\gamma}\frac{dG_m}{dp}w_{nlm}i\sigma_{nlm}Z_{nlm}e^{in\lambda} \quad (7)$$

where Ω is the angular speed of Earth's rotation, g the gravity acceleration, R the gas constant of dry air, γ the static stability parameter, σ_{nlm} the eigenfrequency of Laplace's tidal equation, and Z_{nlm} the geopotential component of the Hough function. Refer to Tanaka (1997) for the detail.

3. RESULTS

Figure 1 compares ω at 500 hPa level on January 28, 1989 for (a) NCEP reanalysis, (b) ECMWF reanalysis, (c) Spectral method, and (d) Normal mode method in this study. This specific day is chosen for the demonstration because an explosive cyclogenesis takes places at the Far East to yield a chain of upward and downward motions. A detailed analysis has been documented for this event by Tanaka and Milkovitch (1990). NCEP reanalysis indicates smooth ω compared with apparently noisy ω by ECMWF. The former has a model resolution of T-62, and the latter T-106. The ω by the spectral method (T-42) in this study is similar to NCEP reanalysis. Yet, the values are extremely large over Tibet and Antarctic. Those of the normal mode method are smoother and no extreme values are found over Tibet. The Hough mode truncations in this study are 26 Rossby modes and 24 gravity modes with 15 zonal wavenumbers.

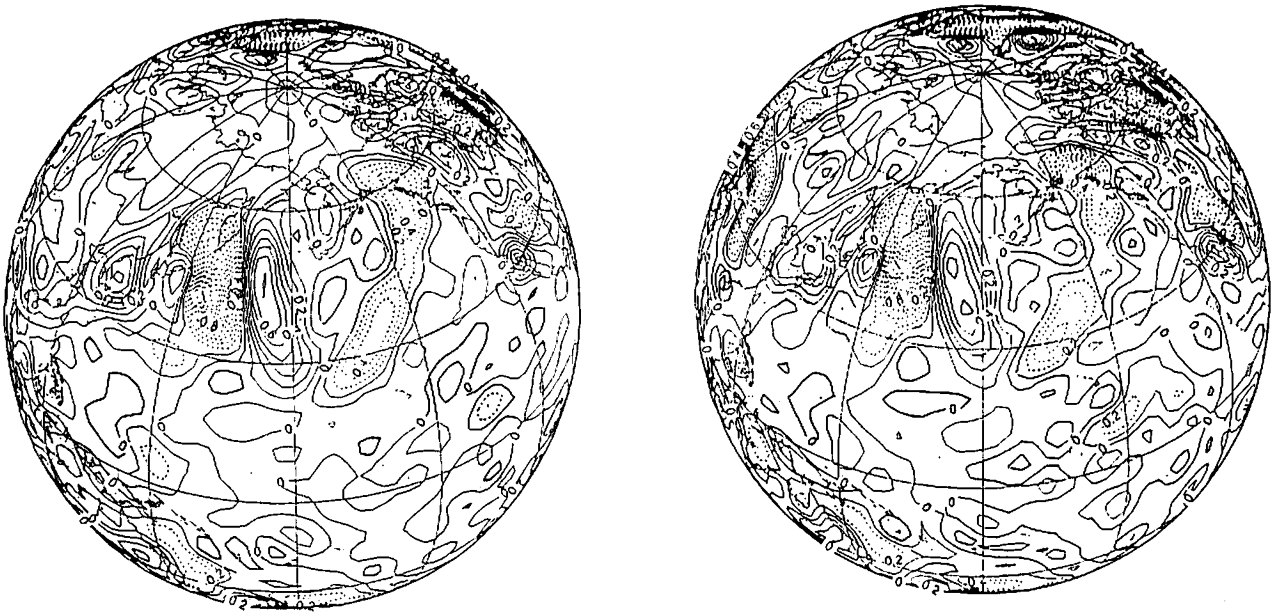
Figure 2 plots vertical profiles of ω at 160°E and 175°E over (a) for 25°N and (b) for 45°N for various scheme (solid line for ECMWF, dotted for NCEP, single dotted dash for spectral, and double dotted dash for normal mode). The differences in the magnitude are substantial. However, the difference for various estimators is within the range of the difference between ECMWF and NCEP reanalyses.

4. SUMMARY AND DISCUSSION

We first realize that the vertical motions

(a) NCEP Omega

(c) NCEP Omega



(b) ECMWF Omega

January 28 89

(d) NCEP Omega

January 28, 1989

Fig. 1 Distribution of ω (Pa/s) at 500 hPa level on January 28, 1989 for (a) NCEP reanalysis, (b) ECMWF reanalysis, (c) Spectral method, and (d) Normal mode method.

provided by NCEP and ECMWF reanalyses and ECMWF TOGA analysis are substantially different; the ω field is noisy in ECMWF, in contrast, it is too smooth in NCEP. The central difference scheme and plane fitting method provide noisy ω field containing small-scale variations as in ECMWF, whereas the spherical harmonic expansion method and normal mode expansion method yield smooth ω field as in NCEP. The difference in the magnitude for various estimators is, however, within the range of the difference between ECMWF and NCEP reanalyses. We need to compare the result with some ground truth to justify the magnitude and distribution of those estimated ω.

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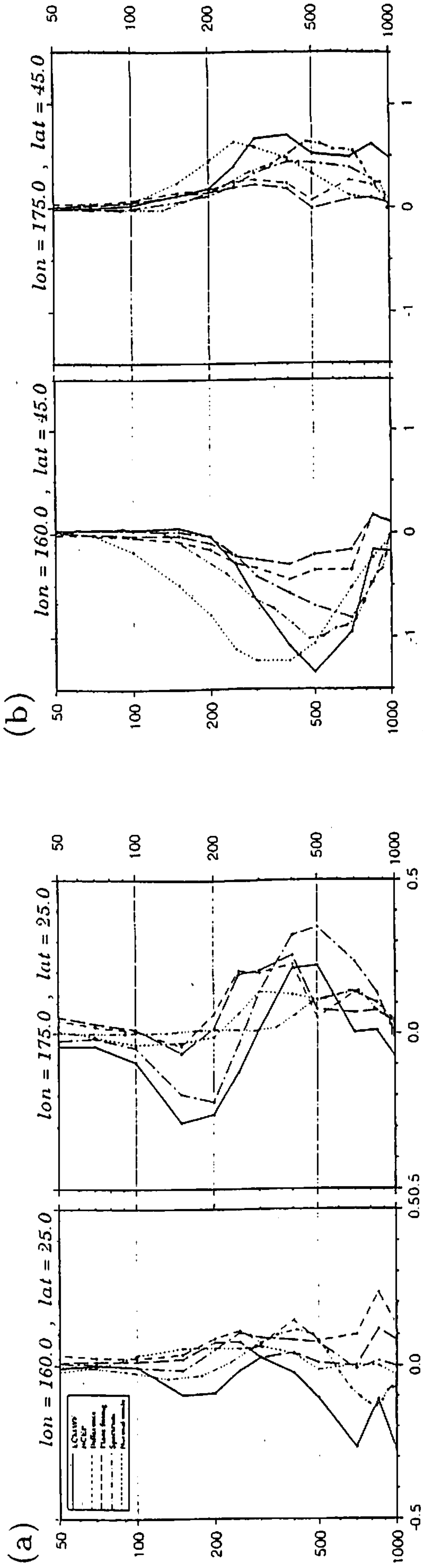

Fig. 2 Vertical profiles of ω (Pa/s) for various scheme at 160°E and 175°E over (a) for 25°N and (b) for 45°N. (solid line for ECMWF, dotted for NCEP, single dotted dash for spectral, and double dotted dash for normal mode).