

# Comparative Energetics of the Old and Recent ECMWF Analyses using the Normal Mode Expansion

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## 1. Introduction

One of the main purposes of FGGE (First GARP Global Experiment) was to improve the medium-range weather forecasting. Improving the data assimilation system as well as the dynamical forecasting models was the main objective of FGGE. The FGGE III-b datasets were produced by two central institutions: the European Center for Medium Range Weather Forecasting (ECMWF) and the Geophysical Fluid Dynamics Laboratory (GFDL). After the release of the FGGE III-b datasets, numerous diagnostic studies have been conducted to evaluate and compare the data assimilation systems of different institutions (e.g., Kung and Tanaka 1983; Lau 1984; Rosen et al. 1985; Holopainen and Fortner 1986; Paegle et al. 1986; Bengtsson et al. 1982; Hollingsworth et al. 1985).

By the results from those comparative studies, the characteristics of different FGGE datasets by different institutions became apparent. It was noted that the GFDL analysis of the FGGE data is noisy containing excessive small-scale structures and is unbalanced for the mass and momentum fields. Arpe et al. (1985) noted that the ECMWF analysis was more faithful to the available data than the GFDL analysis in that the analyzed data by the ECMWF better fit to the observation. The poor fit in the GFDL is, to a large extent, attributed to the small-scale irregularities due to the continuous data insertion. On the other hand, the ECMWF analysis is criticized to have major defects in tropical data. The vertical velocity is weaker in the ECMWF than in the GFDL (Kung and Tanaka 1983), and the divergent wind is damped by a factor of two due to the adiabatic version of the nonlinear normal mode initialization (NLNMI) (see Paegle et al. 1986).

After the production of the original FGGE analysis, the ECMWF has undergone many improvements in the assimilation and initialization technique. The day-to-day operational experience has provided invaluable information on the performance of the system. There has been a continuous process of modifications and improvements, including a diabatic version of NLNMI. With these accumulated knowledges about the defects in the former analysis methods, both of the ECMWF and GFDL conducted the FGGE III-b *re-analyses* based on the improved analysis techniques.

The main changes of the ECMWF *re-analysis* are summarized by Uppala (1986). The results from the new scheme show an intensified divergent and convergent winds in low-latitudes and a better fit of the analysis field with the observed data (see Uppala 1986). Although the FGGE *re-analysis* for the ECMWF was released for public in the mid '80s, the completion and the public release for the GFDL *re-analysis* was not achieved until quite recent in '90s. The revised assimilation

system for the GFDL is described by Stern and Ploshay (1992). The major improvement is summarized by Ploshay et al. (1992) comparing the original and *re-analyses* of FGGE III-b data in reference to those by the ECMWF. It is noted by Stern and Ploshay (1992) and Ploshay et al. (1992) that the new assimilation system shows a significant reduction in the level of noise, improved consistency between the mass and momentum analyses; and a better fit of the analysis field to observations.

In order to assess the achievement of the FGGE project, especially for the improvement in the data assimilation system, Tanaka and Ji (1994) conducted comprehensive comparative studies between the FGGE original analysis and the *re-analysis* produced by the two central institutions of the ECMWF and GFDL, using the normal mode energetics developed by Tanaka (1985) and Tanaka and Kung (1988). The normal mode energetics is an extension of the standard spectral energetics by Saltzman (1957) to three-dimensional (3-D) spectral domain. The atmospheric energy spectrum is plotted as a function of the 3-D scale index, and the nonlinear energy interactions among the different scales of motion are examined. This diagnostic analysis is especially useful in comparing the amount of high-frequency gravity modes which are closely related to the divergent field and are most sensitive to the assimilation technique employed.

With this analysis scheme, Tanaka and Ji showed that the Rossby mode energy levels are virtually same for these FGGE datasets. However, a notable discrepancy is found in the gravity-mode energy spectrum between the ECMWF and GFDL analyses. They showed that the gravity mode energy has increased by 50% for the GFDL *re-analysis* compared with its original analysis, whereas it has decreased by 10% for the ECMWF *re-analysis*. Hence, the discrepancy in the gravity mode energy between the ECMWF and GFDL has grown rather than reduced by the *re-analyses*. Since the FGGE *re-analysis* by the ECMWF is already 10 years old, it is anticipated that the recent ECMWF analysis might have improved substantially during the last 10 years. Hence, another comparative energetics analysis is needed for the old and recent ECMWF analyses using the same normal mode energetics analysis.

The purpose of this study is to compare the energetic characteristics of the latest ECMWF global analysis for winters of 1988/89 and 1993/94 with those of the FGGE *re-analyses*. A special attention is paid to the energy spectra of the Rossby and gravity modes over the 3-D scale index. The difference in the high-frequency gravity mode energy between the old and recent ECMWF analyses is presented. Finally, an assessment is made of how the data analysis is modified by the latest assimilation systems.

## 2. Data and analysis scheme

The FGGE re-analyses by the ECMWF and GFDL for the Special Observing Period 1 (SOP-1) and the ECMWF TOGA global analysis are obtained from the National Center for Atmospheric Research (NCAR). Twice daily (0000 and 1200 UT) meteorological variables of horizontal wind vector  $V = (u, v)$ , vertical p-velocity  $\omega (= dp/dt)$ , temperature  $T$ , geopotential height  $Z$ , and relative humidity  $R$  are defined at 12 mandatory vertical levels of 1000, 850, 700, 500, 400, 300, 250, 200, 150, 100, 70, and 50 hPa. For the GFDL dataset, the top two vertical levels are given at 50 and 30 hPa rather than the aforementioned levels. The ECMWF TOGA global analysis during the winter three months from December to February are examined for 1988/89 and 1993/94. Those are uninitialized gridded data.

The analysis scheme of the normal mode energetics has been detailed in Tanaka (1985) and Tanaka and Kung (1988). The brief description of the scheme is summarized here. First, the vertical structure functions and the Hough harmonics are computed, using a reference state of the global mean temperature. Applied to a sequence of vertical, Fourier, and Hough transforms, the primitive equations become a system of ordinary, dimensionless differential equations in terms of the spectral expansion coefficients in the 3-D spectral domain:

$$\frac{d}{dt} w_{nlm} + i\sigma_{nlm} w_{nlm} = b_{nlm} + c_{nlm} + d_{nlm}, \quad (1)$$

where the complex variables of  $w_{nlm}$ ,  $b_{nlm}$ ,  $c_{nlm}$ , and  $d_{nlm}$  represent respectively the expansion coefficients for a dependent variable vector ( $u, v, Z$ ), nonlinear term vector for the momentum field, for the mass field, and for the diabatic processes. The symbol  $\sigma_{nlm}$  is a dimensionless eigenfrequency obtained as a solution of Laplace's tidal equation with a basic state at rest. The subscripts  $n, l, m$  are for the zonal wavenumber, meridional index, and vertical index, respectively. The vertical indices  $m=0$  and  $m \neq 0$  represent barotropic (external) and baroclinic (internal) modes, respectively.

An element of a total energy norm is simply defined by an absolute value of the expansion coefficient:

$$E_{nlm} = \frac{1}{2} p_s h_m |w_{nlm}|^2, \quad (2)$$

where the dimensional factors  $p_s$  and  $h_m$  are respectively a constant pressure near the surface and an equivalent height of  $m$ -th vertical index. The corresponding energy balance equation is given by differentiating (2) with respect to time and substituting (1) in the time derivatives:

$$\frac{d}{dt} E_{nlm} = B_{nlm} + C_{nlm} + D_{nlm}. \quad (3)$$

According to Eq. (3), the time change of the element of the total energy,  $E_{nlm}$ , is caused by the three terms in the right hand side: i) nonlinear mode-mode interaction of kinetic energy  $B_{nlm}$ , ii) that of available potential energy  $C_{nlm}$ , and iii) a net energy source and sink due to the diabatic process  $D_{nlm}$ , which includes both generation and dissipation of energy. The diabatic process is evaluated as the residual balance of Eq. (3).

Results of the energetics analysis presented in this study are of the global variables. The energy spectrum expressed as a function of  $\sigma_{nlm}$  represents the 3-D spectral energy distribution. By summing the energy terms with respect to all indices but  $n$ , the result describes the zonal energy spectrum as in Saltzman (1957). It is unique in the normal mode energetics that the energy summation can be separately taken for Rossby and gravity modes. Therefore, we can examine the energy levels and the interactions separated in Rossby and gravity modes.

Fig. 1. Spectral distributions of atmospheric total energy ( $\text{Jm}^{-2}$ ) in the frequency domain for ECMWF (left) and GFDL (right) re-analyses for barotropic component ( $m=0$ ). The dimensionless eigenfrequencies  $|\sigma_l|$  in Eq. (1) are used in the abscissa to represent the meridional scale of atmospheric motions. Rossby and gravity modes are plotted in the low- and high-frequency ranges, respectively, for zonal wavenumbers  $n=1$  to 6 (after Tanaka and Ji, 1994).

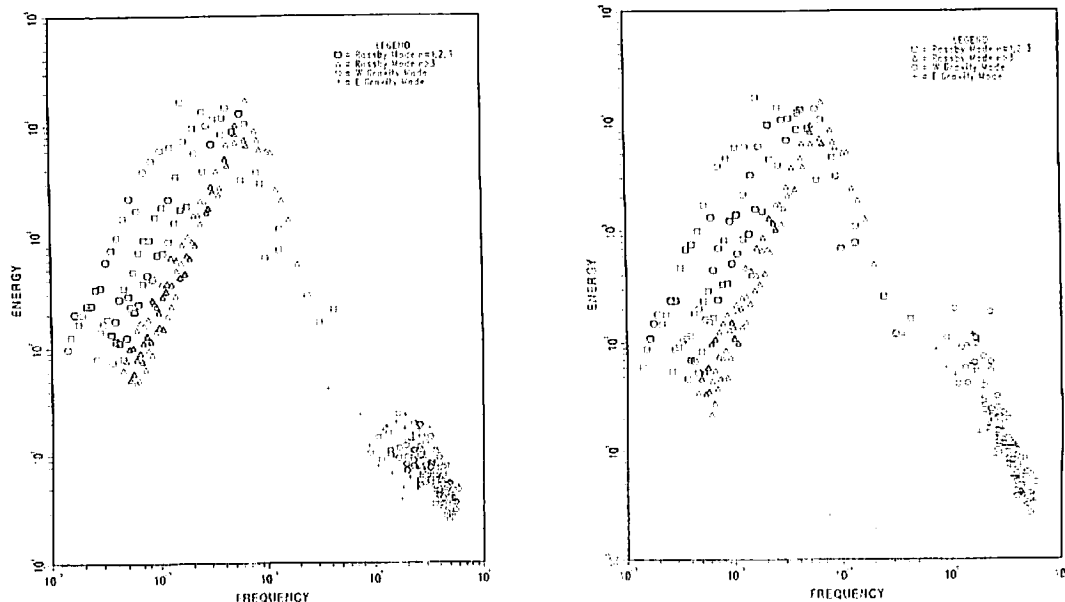
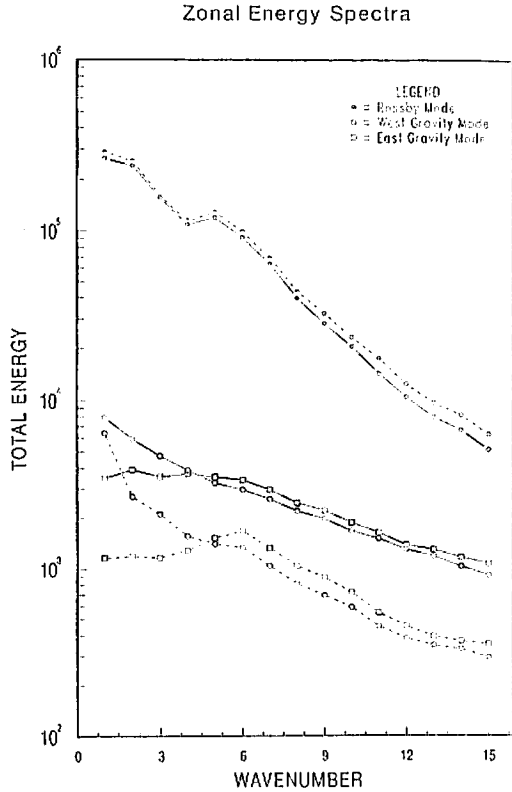
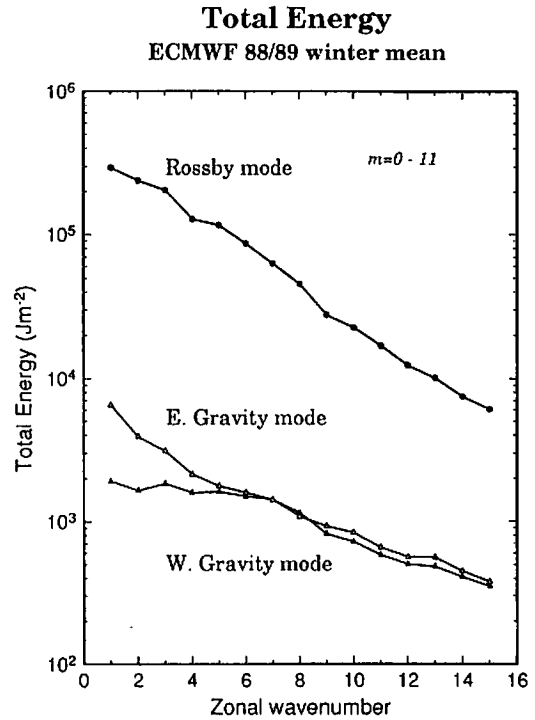


Fig. 3. Zonal energy spectra ( $\text{Jm}^{-2}$ ) over  $n=1-15$  for ECMWF (dashed) and GFDL (solid) re-analyses (after Tanaka and Ji, 1994). Dots, squares, and circles denote Rossby modes and westward and eastward gravity modes, respectively.



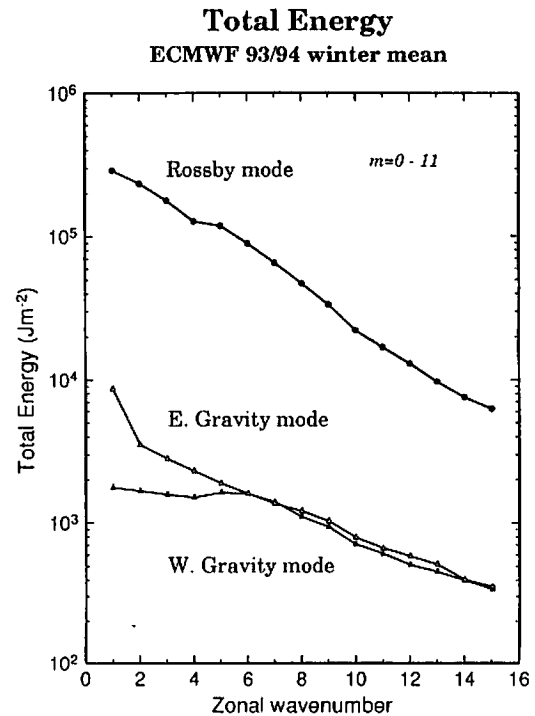
using another dataset produced by different assimilation technique.

Fig. 4. As in Fig. 3 but for the winters of 1988/89 and 1993/94.



Finally, the nonlinear energy transformations of kinetic energy  $B_{nlm}$ , available potential energy  $C_{nlm}$ , and the diabatic process  $D_{nlm}$  are presented for the recent ECMWF analyses in Fig. 5. The results are compared for Rossby, westward gravity, and eastward gravity components. It is evident that the basic characteristics of the energy transformation are determined by the Rossby modes, and the contributions from the gravity modes are relatively small. The large positive values of  $C_{nlm}$  indicate the energy supply from the zonal available potential energy to the individual zonal wave. The negative values of  $B_{nlm}$  show the barotropic conversions from waves to zonal motion. The results are consistent with the traditional zonal spectral energetics by Saltzman. It is important to mention that the zonal wavenumber 1 obtains kinetic energy by the nonlinear interactions for 1993/94 as originally discovered by Saltzman.

Figure 5 presents the energy interactions not only for the Rossby modes, but also for the gravity modes. Although the magnitude is substantially small compared with that of Rossby modes, the gravity mode energy interactions show interesting features. The westward gravity modes obtain energy by means of the baroclinic conversion, and lose the energy by the barotropic conversion as in the case of Rossby modes. In contrast, the eastward gravity modes obtain energy by the barotropic conversion and lose the energy through diabatic process. Those results should be investigated further, comparing



### 3. Results of the analysis

Figure 1 illustrates the energy spectra of the ECMWF and GFDL re-analyses expanded in the 3-D NMFs (see Tanaka and Ji 1994). The energy elements of  $E_{nlm}$  in Eq. (2) are plotted as functions of the scale index  $|\sigma_{nlm}|$  for 50 meridional indices  $l$ , zonal wavenumbers  $n=1-6$ , and vertical indices  $m=0$ . The 50 meridional indices include 26 Rossby modes in the low-frequency side and 24 gravity modes in the high-frequency side. These two kinds of distinct modes are connected by the mixed Rossby-gravity modes (i.e., the largest-scale Rossby modes) which have the highest-frequencies among the Rossby modes. The largest-scale gravity modes are referred to as Kelvin modes, which appear at the lowest-frequency gravity modes.

As is shown by Tanaka (1985), clear energy peaks appear at the intermediate meridional indices within the Rossby modes. Separated by these peaks associated with each zonal wavenumber, the energy levels decrease toward the Rossby mode regime at the low-frequency side indicating a  $-3$  power law and toward the gravity mode regime at the high-frequency side.

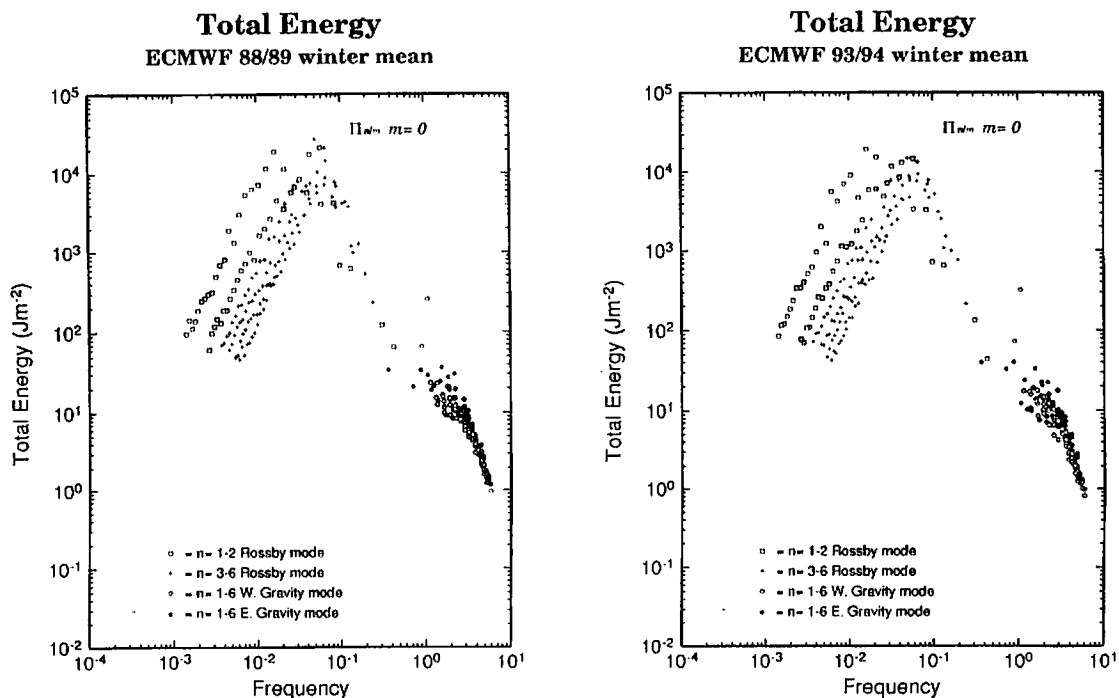
The comparison of the external modes of  $m=0$  for the ECMWF and GFDL re-analyses shows similar energy spectra for the Rossby modes. However, a marked discrepancy is found in the gravity-mode energy spectrum. The gravity mode spectrum for the GFDL exhibits a  $-3/5$  power law with respect to the scale index  $\sigma_{nlm}$ . The larger the modal scale is, the higher the energy level is. The largest-scale gravity modes indicate their energy levels higher than  $200 \text{ Jm}^{-2}$ . In contrast, the gravity mode energy levels for the ECMWF are lumped together near  $10 \text{ Jm}^{-2}$ , and the power law is not recognized. The low-frequency (i.e., large-scale) gravity modes are obviously damped for the ECMWF. The energy level is one order of magnitude lower than the results of the GFDL.

Similar energy spectra are illustrated in Figure 2 for the winters of 1988/89 and 1993/94. For these recent ECMWF analyses, we can notice that the gravity mode energy spectrum has modified so that the spectral distribution becomes similar to that of GFDL. The largest energy levels for the westward gravity modes have increased, exceeding  $300 \text{ Jm}^{-2}$  which is ten times larger than that of the re-analysis. The energy spectrum now indicates some power law characteristics.

Figure 3 illustrates zonal energy spectra for re-analyses by the ECMWF (dashed lines) and GFDL (solid lines) for  $n=1-15$ . The results are compared separately for Rossby and gravity modes. The Rossby mode energy level for the ECMWF is slightly higher than that for the GFDL, but the difference is relatively minor. The gravity mode energy level is, however, substantially different between these two re-analyses. The energy levels by the GFDL are consistently higher, especially for large  $n$ . For planetary waves of  $n \leq 4$ , the eastward gravity modes are greater than the westward gravity modes due to the dominant Kelvin waves at the baroclinic components. The spectral features are basically same for the FGGE original analysis as documented in Tanaka and Ji (1994).

Figure 4 compares the zonal energy spectra for the winters of 1988/89 and 1993/94. Rossby mode energy spectra are similar for all cases compared with the re-analysis. The interannual variability seems to be quite small for the Rossby components. On the other hand, we can find by this result that the gravity mode energy spectra in the recent ECMWF analysis are still lower than that of the GFDL reanalysis. It should be noted that we can not conclude that one is superior to the other because we do not know the true energy level for the gravity modes in the atmosphere. Nevertheless, indicating the evident differences by different analysis techniques should be meaningful.

Fig. 2. As in Fig. 1 but for the winters of 1988/89 and 1993/94.



#### 4. Concluding remarks

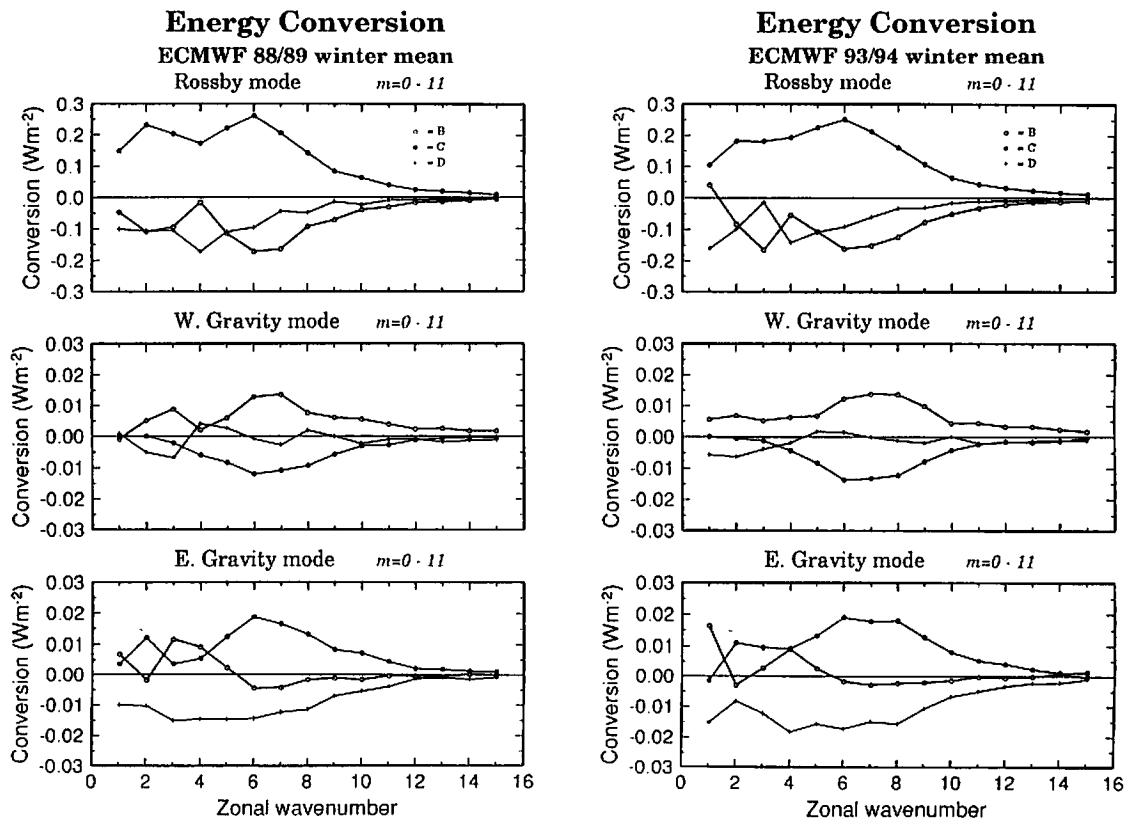
This study conducted the energetics diagnosis of the latest ECMWF global analysis for winters of 1988/89 and 1993/94, and the results are compared with the FGGE III-b re-analyses produced by the ECMWF and GFDL. The energy levels of the atmospheric Rossby and gravity modes are examined by means of the normal mode energetics scheme.

As the result, we confirmed that the Rossby mode energy levels are virtually same for these FGGE datasets and recent ECMWF analyses. The Rossby modes appear to be less sensitive to the analysis technique. However, a notable discrepancy is found in the gravity-mode energy spectrum between the ECMWF and GFDL analyses. For the external modes, the GFDL analyses exhibit a -3/5 power law in the gravity mode energy spectrum with respect to the 3-D scale index  $\sigma_{nm}$ . The gravity mode energy levels for the ECMWF re-analysis are, however, lumped together at a very low level, and the power law is not recognized. We find in this study that the gravity mode energy spectrum in the recent ECMWF has modified so that the low-frequency (i.e., largest-scale) eddy gravity modes are significantly amplified to reveal the power law as seen in the GFDL data. However, the energy level of the gravity modes are still substantially lower than that of the GFDL analysis. Since we do not know the true energy spectrum for the gravity modes, further comparative normal mode energetics analysis is desired for other datasets, such as the NMC and JMA global analyses.

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Fig. 5. Nonlinear energy transformations of kinetic energy  $B_{nlm}$ , available potential energy  $C_{nlm}$ , and diabatic process  $D_{nlm}$  over the zonal wavenumber  $n=1-15$  for the winters of 1988/89 and 1993/94. Top: Rossby modes; middle: westward gravity modes; and bottom: eastward gravity modes.



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