

Time Spectral Analysis for the Natural Variability of the Barotropic Model Atmosphere

H. L. Tanaka and Kazuo Kimura
Institute of Geoscience
University of Tsukuba, Japan

Keywords: climate model, climate change, natural variability, time spectrum

1 Scientific Background

Climate variation in nature is composed of two components: one is caused by unusual external forcing and the other is the inherent natural variability which occurs even under the fixed boundary conditions. A detailed analysis of climate variability is an important research subject to understand the issue in the global change. Occurrence of short term abnormal weather may be regarded as a signal of the climate variation superimposed on the natural variation.

Recent global warming is anticipated to have been caused mostly by the increased CO_2 , which acts as the external forcing of the atmosphere. It is important to know the quantitative magnitude of the recent warming induced by the greenhouse gases such as CO_2 . If the anticipated warming is caused mostly by these greenhouse gases, some immediate political action has to be taken to reduce the greenhouse gases. However, if the recent warming is just due to the natural trend or natural variability of the chaotic atmosphere which has no relation to the increased greenhouse gases, there is no need to worry about the release of these greenhouse gases into the atmosphere. Therefore, the first step to assess the recent climate variation is to separate the variability into the forced trend due to the increased greenhouse gases and a natural variability of the atmosphere superimposed on the forced trend.

Atmospheric general circulation models under fixed boundary conditions, such as the fixed sea surface temperature (SST), indicate their own natural variability determined by the characteristic physical processes included in the models. Likewise, coupled atmosphere-ocean models and more complicated climate system models would indicate their own natural variability under the fixed wider class of the boundary conditions, such as the type of gas constituent and astronomical parameters. The process to determine the magnitude of the natural variability depends highly on the complexity of the model. However, it is not clear how the natural variability depends on the complexity and the internal nonlinear structure of the model. Even a simple barotropic model with an energy source and sink can show its characteristic natural variability. Hence, it is an interesting subject to evaluate the magnitude of the natural variability for different climate models and find some rules between the model complexity and the induced natural variability.

2 Objective

The objective of this study is to find the typical magnitude of the natural variability of a model atmosphere by a long term integration (1000 years) of a simple barotropic model under a fixed boundary condition. The results will be compared with the natural variability of other climate models.

3 Model description

A 3-D spectral representation of primitive equations may be written by the following general form after a suitable diagonalization of the linear terms:

$$\frac{dw_i}{d\tau} + i\sigma_i w_i = -i \sum_{jk} r_{ijk} w_j w_k + f_i, \quad i = 1, 2, 3, \dots$$

where w_i and f_i represent the spectral expansion coefficients of the dependent variables and external forcing, respectively. The symbol σ_i denotes the eigenfrequency of the normal mode in a resting atmosphere, and r_{ijk} is the interaction coefficient for nonlinear wave-wave interactions.

In the 3-D spectral representation, the vertical expansion basis functions may be divided in barotropic and baroclinic components. In this study, we attempt to construct a spectral barotropic model, using only the barotropic components of w_i . The spectral equation for such a barotropic model has the same form as for the baroclinic model, except the fact that the barotropic-baroclinic interactions should be included formally in f_i .

In this study, we consider the next forcing:

$$f_i = (BC)_i + (DF)_i + (ZS)_i + (VP)_i,$$

where $(BC)_i$ represents the baroclinic instability, $(DF)_i$ the biharmonic diffusion, $(ZS)_i$ the zonal surface stress, and $(VP)_i$ the vertical propagation of planetary waves. The unique energy source of the model is $(BC)_i$, and the rest of the three physical processes are the energy sinks in this model. The nonlinear interaction is designated as $(NL)_i$. Refer to Tanaka (1991) for the detail of the model description.

4 Results

We integrated the simple spectral barotropic model for 1000 years under a fixed boundary condition. Figure 1a to 1d illustrate the time series of the total energy of the model atmosphere for 1, 10, 100, and 1000 years, respectively. The model output is stored for every 24 hours. Those time series in Figs 1 are the plots for daily value, 10-day, 100-day, and 1000-day averages, respectively. Namely, every panel of the time series contains 365 points of data.

Figure 1a shows a smooth variation of energy level starting from about $11 \times 10^5 \text{ Jm}^{-2}$. The energy increases to $17 \times 10^5 \text{ Jm}^{-2}$ after about 20 days and has reached to an equilibrium, fluctuating the mean level. This time series seems to contain a low-frequency variability with a period of approximately 50 days.

Figure 1b is the time series of the 10-day mean of the same energy variation for 10 years. We can see the initial increase at the first 20 days at the left-most edge of the time series. The energy level varies within a range from 14 to $22 \times 10^5 \text{ Jm}^{-2}$. This range characterizes the magnitude of the natural variability of the model atmosphere.

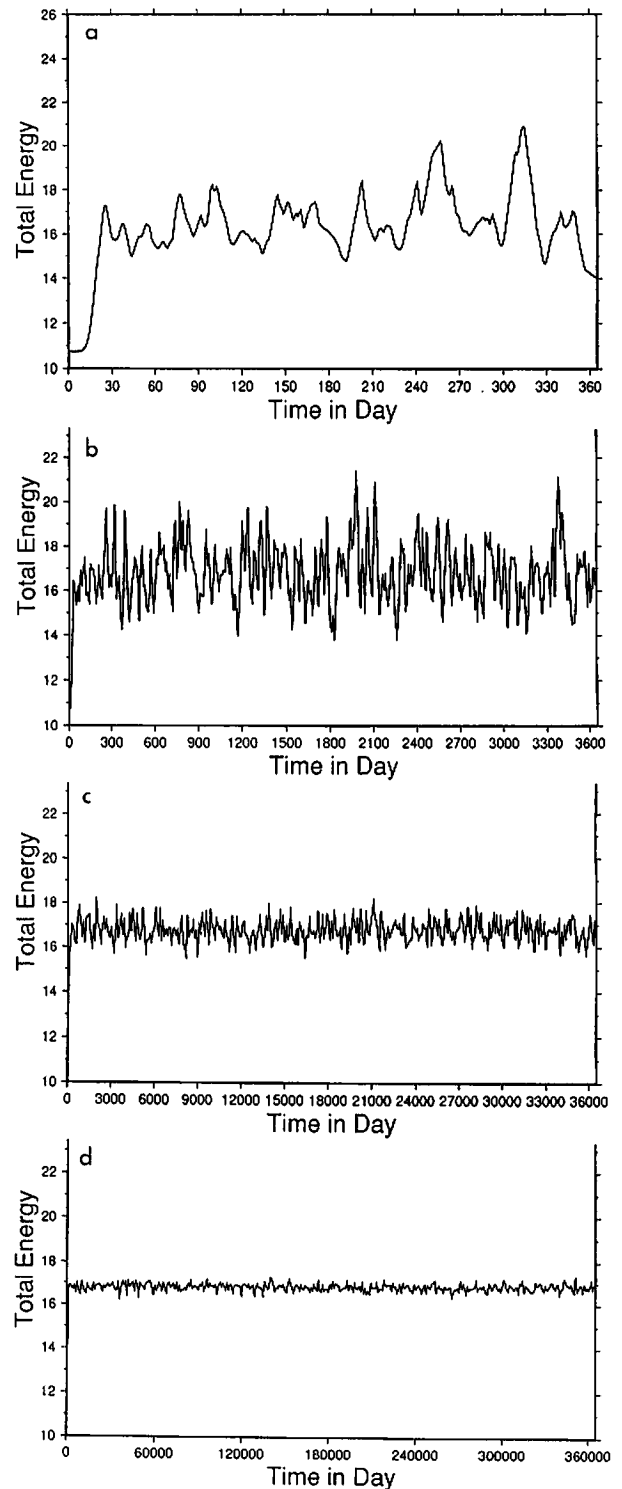
Figure 1c is for the 100-day mean time series during 100 years. Since the 100-day mean filters out most of the dominant oscillations, the range of the natural variability is reduced substantially compared with the 10-day mean time series in Fig. 1b. Similarly, Fig. 1d is for the 1000-day mean time series during 1000 year integration period. The range of the natural variation has reduced in this plot to about $1 \times 10^5 \text{ Jm}^{-2}$. This time series differs substantially from that of Fig.1a in its characteristics of variation. We find practically no ultra-low-frequency variability at this time scale beyond the 50 day period.

Figure 2 illustrates the power spectrum of the time series of the total energy for the 1000-year integration. The power spectrum is estimated by the FFT routines and is smoothed by averaging 64 term of the original power spectrum. The frequency resolution of the spectrum is about $1/(10\text{-years})$ which is still fine enough to argue the detailed spectral properties.

The result clearly separates the two characteristic spectral slopes of the model atmosphere. It is obvious that the spectrum is white for the period beyond 50 days. In other word, the random variable has no memory of the past for the variation beyond the 50-day period. In contrast, the spectrum is red for the period shorter than 50 days. Namely, the random variable remembers its own past to some extent for the variation shorter than 50-day period. Interestingly, the spectral slope for the red noise obeys approximately the -3 power of the frequency. There is an energy source into the system at the frequency range near $1/(5\text{-day})$ due to the parameterized baroclinic instability. Separated by this energy source, the spectral slope appears to change to -4 power of the frequency at the higher frequency range over $1/(5\text{-day})$ to $1/(2\text{-day})$.

The red noise spectrum extends over the range from $1/(50\text{-day})$ to $1/(2\text{-day})$. Although there is no apparent spectral peak near the frequency at $1/(50\text{-day})$, we can observe the dominant variation with the time scale of about 50 days as in Fig. 1a due to the fact of the dominant spectral power at this range relative to the higher frequency range of the red noise.

Figure 1 Time Series



5 Summary

The results of the 1000 year time integration of a simple barotropic model with an energy source from the parameterized baroclinic instability are summarized as the following three points:

1. The total energy of the model atmosphere shows natural variation in the range from $14\text{--}22 \times 10^5 \text{ Jm}^{-2}$ for the long-term average of $17 \times 10^5 \text{ Jm}^{-2}$.
2. The time spectrum of the model atmosphere is characterized as white noise for the low-frequency variability with its period longer than 50 days. In contrast, the spectrum is characterized as red noise for the period shorter than 50 days.
3. It is found in this study that the red noise spectrum over the spectral range for $1/(50\text{-day})$ to $1/(5\text{-day})$ obeys a characteristic -3 power law. Since there is a unique energy source into the system near the frequency at $1/(5\text{-day})$ in terms of the baroclinic instability of synoptic disturbances, we can expect a reverse energy cascade from higher- to lower-frequency ranges associated with the -3 power law of the power spectrum.

6 Acknowledgments

We appreciate Dr. S. Mitsumoto for his kind arrangement in using the computing facility at the NIES.

7 References

1. H.L. Tanaka, *Mon. Wea. Rev.*, **119**, 2919-2935 (1991)

Figure 2
Power Spectrum
Total Energy

