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A Parameterization of Baroclinic Instability in a Barotropic Model

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1. INTRODUCTION

In a framework of a barotropic and baroclinic decomposition of atmospheric motions, the energy flows from zonal baroclinic components via eddy baroclinic components to eddy barotropic components. The eddy barotropic energy is further transferred to zonal barotropic components by means of the inverse energy cascade (see Wiin-Nielsen 1962; Tanaka 1991). The energy supply into the barotropic atmosphere is dominated by a process of baroclinic instability at the zonal wavenumbers about 5 to 10. Therefore, parameterizing baroclinic instability is essential in constructing a barotropic general circulation model with energy sources and sinks.

In this study, the parameterization of the baroclinic instability by Tanaka (1991) is examined with an expanded barotropic spectral model. The model resolution now corresponds to R20, but is symmetric about the equator, containing only the Rossby modes. The energy levels and energy flows induced by the parameterized baroclinic instability are analyzed for this finer resolution models.

2. DESCRIPTION OF THE BAROTROPIC MODEL

A 3-D spectral representation of primitive equations may be written by the following general form after a suitable diagonalization of the linear terms:

$$\frac{dw_i}{d\tau} + i\sigma_i w_i = -i\sum_{jk} r_{ijk} w_j w_k + f_i, \quad i = 1,2,3,\ldots \quad (1)$$

where w_i and f_i represent the spectral expansion coefficients of the dependent variables and external forcing, respectively. The symbol σ_i denotes the eigenfrequency of the normal mode in a resting atmosphere, and r_{ijk} is the interaction coefficient for nonlinear wave-wave interactions.

In the 3-D spectral representation, the vertical expansion basis functions may be divided in barotropic and baroclinic components. In this study, we attempt to construct a spectral barotropic model, using only the barotropic components of w_i. The spectral equations for such a barotropic model has the same form as (1) except the fact that the barotropic-baroclinic interactions should be included formally in f_i. In this study, we consider the next forcing:

$$f_i = (BC)_i + (DF)_i + (ZS)_i + (VP)_i, \quad (2)$$

where $(BC)_i$ represents the baroclinic instability, $(DF)_i$ the biharmonic diffusion, $(ZS)_i$ the zonal surface stress, and $(VP)_i$ the vertical propagation of planetary waves. The unique energy source of the model is BC_i and the rest of the three physical processes are the energy sinks in this model. The nonlinear interactions in (1) is designated as $(NL)_i$. Refer to Tanaka (1991) for the detail of the model descriptions.

3. PARAMETERIZATION OF BAROCLINIC INSTABILITY

The parameterization for $(BC)_i$ is based on an orthogonal projection of the barotropic spectral coefficients $w_i(t)$ onto the eigenvector ξ of the most unstable mode which is solved linearly for a given baroclinic zonal basic state.

$$w_i(t) = a(t)\xi_0 + \epsilon(t), \quad (3)$$
$$a(t) = \xi_0^H w_i, \quad (4)$$
$$(BC)_i = -i\nu a(t)\xi_0, \quad (5)$$

where the superscript H denotes the complex conjugate transpose. The subscript 0 for ξ_0 stands for the barotropic component of ξ. The projected part is then amplified toward the unstable direction in the phase space by the amount of the growth rate (eigenvalue ν) of the projected unstable mode.

This parameterization works accurately for small amplitude synoptic disturbances in the linear framework. For instance, an infinitesimal white noise of the initial state is amplified toward the most unstable direction along with the growth rate of the linear theory. The exponential growth at this stage is correctly represented by the orthogonal projection because the spectral coefficients of the model are approximately parallel to the prescribed eigenvector of the unstable mode. As the growing mode reaches to a finite amplitude, the nonlinearity of the wave-wave interactions begins to deviate the spectral coefficients from the unstable direction. At this stage, the parameterized growth is imposed only on the fraction of the spectral coefficients, which is parallel to the unstable mode. The amplification due to the baroclinic instability would therefore vanish if the spectral coefficients are completely orthogonal to the eigenvector of the unstable mode.

4. RESULTS

The barotropic spectral model (1) is integrated for

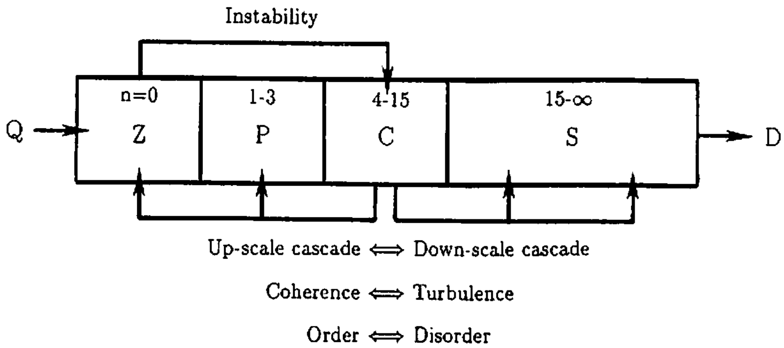

Figure 1. Schematic energy flow in the zonal wavenumber domain. Q and D represents the differential heating and energy dissipation.

400 days starting from small disturbances superimposed on a zonal flow. Table 1 lists the mean total energy $E(n)$, nonlinear interaction $NL(n)$, energy supply due to the baroclinic instability $BC(n)$, diffusion $DF(n)$, vertical propagation $VP(n)$, and zonal surface stress $ZS(n)$ as functions of the zonal wavenumber n.

According to the results, the energy input by the baroclinic instability ultimately balances with the energy scattering due to the nonlinear wave-wave interactions, $NL(n)$, toward the short waves and also to zonal motions. The former is characterized as downscale energy cascade, whereas the latter is regarded as upscale energy cascade. The important role of the nonlinear interactions is to transfer the energy from the source to the sink, which is the characteristic of turbulence.

5. CONCLUDING REMARKS

Present parameterization of the baroclinic instability contains no free parameters. Despite this fact, the mean energy levels are equilibriated at the comparable level as observed in the barotropic component of the atmosphere. The equilibrium is attained in the model by the balance between the linear excitation of $(BC)_i$ and the nonlinear scattering of $(NL)_i$ toward the energy sink of $(DF)_i + (ZS)_i + (VP)_i$.

The energy flows in the zonal wavenumber domain of the present model is summarized in Figure 1. The energy supply due to the differential heating Q at n=0 is transferred to synptic-scale disturbances induced by the baroclinic instability. The accumulated energy then cascades down to smaller scale eddies, whereas a part of the energy cascades up to the planetary waves and the zonal flow within the barotropic atmosphere. The energy flows agree reasonably well with observations (e.g., Saltzman 1970). The results suggest that the present parameterization of the baroclinic instability is useful for wide applications of the simple barotropic models.

It is interesting to see the clear contrast between the downscale and upscale cascades: the downscale cascade causes disorder and turbulence while the upscale cascade creates order and coherence. The jet stream and blocking system in the atmosphere are clearly maintained by such upscale energy cascades as discussed in Tanaka (1991).

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Table 1. Gross energy budget in the wavenumber domain for 101-400 days mean. Refer to the text for definitions of energetic terms. Energy is in 10^2Jm^{-2}, and energy conversion is in 10^{-3}Wm^{-2}.

n	$E(n)$	$NL(n)$	$BC(n)$	$DF(n)$	$ZS(n)+VP(n)$
0	11842	653	0	-14	-630
1	1810	43	21	-41	-24
2	1066	33	26	-36	-15
3	798	25	25	-35	-13
4	810	0	47	-35	-9
5	603	-42	81	-39	-3
6	761	-268	328	-50	0
7	721	-405	482	-58	0
8	319	-125	184	-41	0
9	149	-32	74	-30	0
10	84	7	28	-23	0
11	52	15	10	-18	0
12	40	19	0	-18	0
13	26	17	0	-14	0
14	18	17	0	-13	0
15	13	14	0	-12	0
16	9	12	0	-10	0
17	6	11	0	-9	0
18	5	9	0	-8	0
19	4	9	0	-8	0
20	3	9	0	-7	0