

## Comparative Energetics of FGGE Reanalyses using the Normal Mode Expansion

by  
Hiroshi L. Tanaka  
*Institute of Geoscience  
University of Tsukuba, Japan*

and  
Qiang Ji  
*Geophysical Institute  
University of Alaska Fairbanks, U.S.A.*

### 1 Introduction

One of the main purposes of FGGE (First GARP Global Experiment) was to improve the medium-range forecasting. The FGGE III-b datasets were produced by two central institutions: the European Center for Medium Range Weather Forecasting (ECMWF) and the Geophysical Fluid Dynamics Laboratory (GFDL). After the release of the FGGE III-b datasets, numerous diagnostic studies have been conducted to evaluate and compare the data assimilation systems of different institutions (e.g., Kung and Tanaka 1983; Lau 1984; Paegle et al. 1986). With the results from those comparative studies, understanding of the distinct characteristics of different FGGE datasets has increased.

It is noted that the GFDL analysis of the FGGE data is noisy containing significant small-scale structure, and the mass and momentum fields are unbalanced. The spurious excitation of the small-scale disturbance may be resulted from the continuous data insertion. On the other hand, the ECMWF analysis is pointed out to have major deficiencies in the tropics. The vertical velocity is weaker in the ECMWF than the GFDL (Kung and Tanaka 1983), and the divergent wind has been damped by a diabatic version of the nonlinear normal mode initialization (Paegle et al., 1986). The imposed 95% constraint toward the geostrophic balance seems to be too restrictive in the tropics.

Since the production of the FGGE original analyses, there has been a continuous process of modifications and improvements. The day-to-day operational experience has provided invaluable information on the performance of the system. With

these accumulated knowledge of the deficiencies in the former data analysis methods, both of the ECMWF and GFDL conducted the FGGE III-b *reanalyses* based on the improved assimilation techniques. Refer to Uppala (1986) for the detail of the ECMWF reanalysis and Stern and Ploshay (1992) for the GFDL.

The purpose of this study is to compare the energetic characteristics of the FGGE reanalyses with those of the FGGE original analyses. The comparison is made of both the ECMWF and GFDL versions of the original and reanalyses.

One of the useful diagnostic tools for that purpose is the normal mode energetics developed by Tanaka and Kung (1988). In the normal mode energetics, observed atmospheric data are expanded in the 3-D normal modes for a resting atmosphere which are constructed by a product of the vertical structure functions and Hough harmonics. The eigenfrequency of a 3-D normal mode may be regarded as a 3-D scale index for each of the gravity (first kind) and rotational (second kind) modes due to the intrinsic dispersion relation. The atmospheric energy spectrum is represented as a function of the 3-D scale index, and the nonlinear energy interactions among the different scales of motion are examined.

This diagnostic analysis is especially useful in comparing the amount of high-frequency gravity modes which are closely related to the divergent field and are most sensitive to the assimilation system. Therefore, we have paid a special attention to the energy spectra of the gravity modes over the 3-D scale index. The difference in the high-frequency gravity mode energy between the ECMWF and GFDL analyses is presented. Finally, an assess-

ment is made of how the difference in the original analyses is modified by the new assimilation systems.

## 2 Data and analysis scheme

The FGGE reanalyses by the ECMWF and GFDL for the Special Observing Period 1 (SOP-1) are obtained from the National Center for Atmospheric Research (NCAR). Twice daily (0000 and 1200 UT) meteorological variables of horizontal wind vector  $V = (u, v)$ , vertical p-velocity  $\omega (= dp/dt)$ , temperature  $T$ , geopotential height  $Z$ , and relative humidity  $R$  are defined on  $1.875^\circ$  by  $1.875^\circ$  horizontal grids at 12 mandatory vertical levels of 1000, 850, 700, 500, 400, 300, 250, 200, 150, 100, 70, 50 mb. For the GFDL dataset, the top two vertical levels are given at 50 and 30 mb rather than the aforementioned levels. These FGGE data are interpolated onto  $5^\circ$  longitudinal grids and 60 Gaussian latitudinal grids.

The analysis scheme of the normal mode energetics has been detailed in Tanaka and Kung (1988). The brief description of the scheme is summarized here. First, applied to a sequence of vertical, Fourier, and Hough transforms, the primitive equations become a system of ordinary, dimensionless differential equations in the 3-D spectral domain:

$$\frac{d}{dt}w_{klm} + i\sigma_{klm}w_{klm} = n_{klm} + d_{klm}, \quad (1)$$

where the complex variables of  $w_{klm}$ ,  $n_{klm}$ , and  $d_{klm}$  represent respectively the expansion coefficients for a vector  $(u, v, Z)$ , nonlinear term vector for the momentum and mass fields, and the diabatic processes including friction. The symbol  $i$  represents the imaginary unit, time  $t$  is normalized as  $\tilde{t} = 2\Omega t$  with the angular speed of the earth's rotation  $\Omega$ , and  $\sigma_{klm}$  is a dimensionless eigenfrequency obtained as a solution of Laplace's tidal equation with a basic state at rest. The subscripts  $k, l, m$  are for the zonal wavenumber, meridional index, and vertical index, respectively. The vertical indices  $m=0$  and  $m \neq 0$  represent barotropic (external) and baroclinic (internal) modes, respectively. We have used a total of 50 meridional modes, including 26 Rossby modes and 12 eastward and 12 westward gravity modes.

An element of total energy is simply defined by an energy norm of the system:

$$E_{klm} = \frac{1}{2}p_s h_m |w_{klm}|^2, \quad (2)$$

where the dimensional factors  $p_s$  and  $h_m$  are respectively a constant pressure near the surface and an equivalent height of  $m$ th vertical index. The energy spectrum expressed as a function of  $\sigma_{klm}$  represents the 3-D spectral energy distribution. By summing the energy terms with respect to all indices but  $k$ , the result describes the zonal energy spectrum, while the summations over  $k$  and  $l$  describe the vertical energy spectrum.

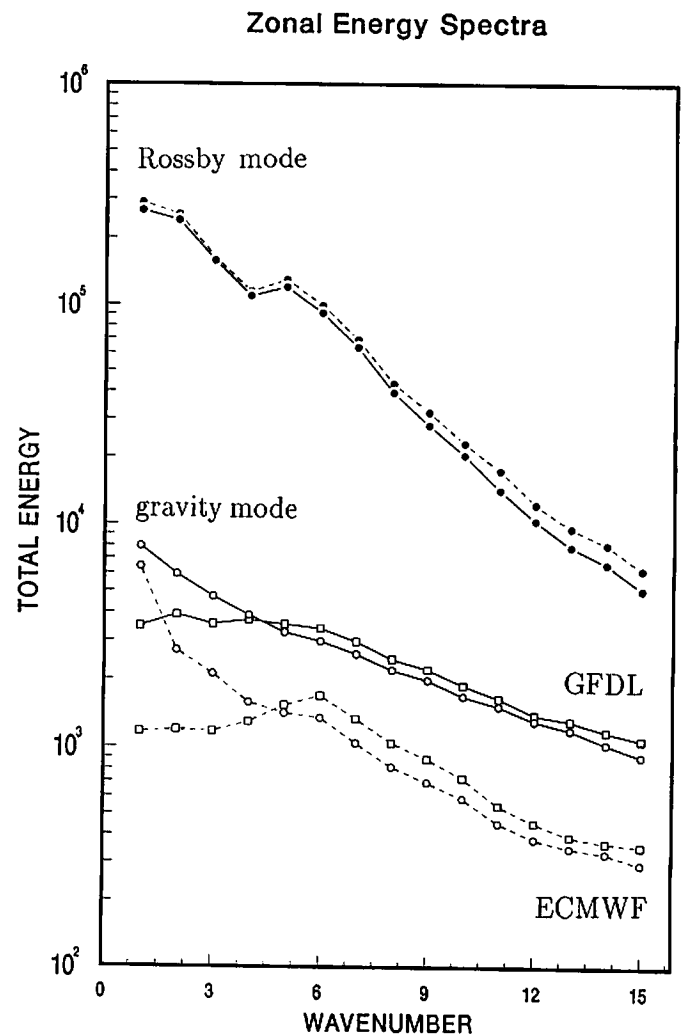


Fig. 1. Zonal energy spectra for the ECMWF (dashed) and GFDL (solid) reanalyses for  $k=1-15$ . Dots, squares, and circles denote Rossby modes and westward and eastward gravity modes.

### 3 Results of the analysis

Figure 1 illustrates zonal energy spectra for  $k=1-15$  as obtained by summing all meridional and vertical indices for Rossby and gravity modes. The Rossby mode energy levels indicate minor difference between the ECMWF (dashed) and GFDL (solid). The gravity mode energy level is, however, substantially different between these two reanalyses. The GFDL analysis is about 4 time larger than the ECMWF analysis at  $k=15$ . For planetary waves of  $k \leq 4$ , the eastward gravity modes are greater than the westward gravity modes due to the dominant Kelvin waves.

Figure 2 shows the energy spectra of the ECMWF and GFDL reanalyses expanded in the 3-D NMFs. The energy elements of  $E_{klm}$  in Eq. (2) are plotted as functions of the scale index  $\sigma_{klm}$  for 50 meridional indices  $l$ , zonal wavenumbers  $k=1-6$ , and vertical indices  $m=0$  (Fig. 2a) and  $m=2$  (Fig. 2b). The difference in the modal energy levels are visually presented by deviation bars. The energy levels for the GFDL reanalysis are plotted with symbols, and the deviation bars are extended toward the the ECMWF reanalysis without symbols. The comparison of the external modes of  $m=0$  for the two reanalyses shows similar energy spectra for the Rossby modes. However, the discrepancy is evident in the gravity-mode energy spectrum. The gravity mode energy levels for the ECMWF are lumped together near  $10 \text{ Jm}^{-2}$ , and a power law is not recognized. The low-frequency gravity modes are significantly damped in the ECMWF; the energy level is one order of magnitude lower than the results of the GFDL reanalysis. Figure 2b compares the same energy spectra, but for an internal mode of  $m=2$ . Again, the Rossby-mode energy spectra are similar for these two reanalyses. It is shown that planetary-scale Kelvin modes dominate the other gravity modes, and the energy levels reach up to  $1000 \text{ Jm}^{-2}$  for these two reanalyses. A discrepancy is found in the high-frequency (i.e., small-scale) gravity modes; the energy level of the GFDL is about one order of magnitude higher than that of the ECMWF for the smallest resolvable-scale modes.

### 4 Concluding remarks

This study conducted the comparative diagnosis of the FGGE III-b reanalyses produced recently by the ECMWF and GFDL. The energy levels of the atmospheric gravity modes are examined by means of the normal mode energetics scheme.

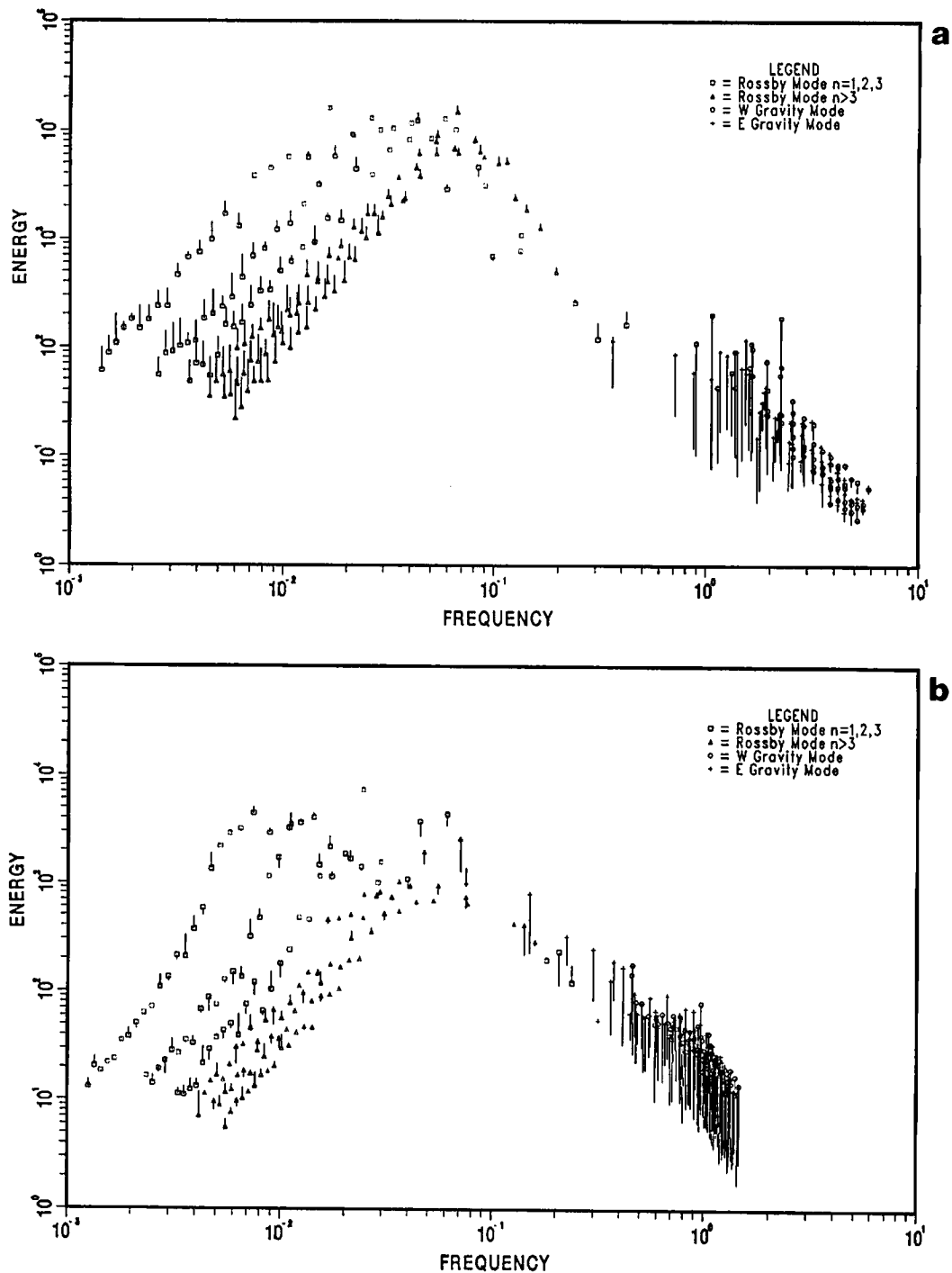
As a result, we confirmed that the Rossby mode energy levels are virtually same for these FGGE datasets. The Rossby modes appear to be less sensitive to the analysis technique. However, a notable discrepancy is found in the gravity-mode energy spectrum between the ECMWF and GFDL analyses. For the external modes, the GFDL analyses exhibit a power law in the gravity mode energy spectrum with respect to the 3-D scale index  $\sigma_{klm}$ . In contrast, the gravity mode energy levels for the ECMWF are lumped together near  $10 \text{ Jm}^{-2}$  level, and the power law is not recognized. In the ECMWF reanalysis, the low-frequency (i.e., largest-scale) external gravity modes are one order of magnitude lower than the results of the GFDL reanalysis. The internal modes are also damped by an order of magnitude at the high-frequency (i.e., small-scale) component, compared with the corresponding GFDL reanalysis.

It is worth noting that the gravity mode energy has increased by 50% for the GFDL reanalysis compared with its original analysis, whereas it has decreased by 10% for the ECMWF reanalysis. Hence, the discrepancy in the gravity modes between the ECMWF and GFDL has grown rather than reduced by the reanalyses. This result was unexpected from the fact of smoother fields in the GFDL reanalysis and intensified Hadley circulation in the ECMWF reanalysis. It is speculated that the GFDL reanalysis contains larger amount of ageostrophic wind as found from the poor fit of the geopotential field to the observed field (see Ploshay et al. 1992).

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**Fig. 2.** Energy spectra of the GFDL (with symbols) and ECMWF (deviation bar) reanalyses for vertical indices  $m=0$  (Fig. 2a) and  $m=2$  (Fig. 2b) as functions of the scale index  $\sigma_{klm}$ .

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