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BLOCKING FORMATIONS BY THE TURBULENT UP-SCALE CASCADE

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1. Introduction

Amplification of low-frequency planetary waves in the troposphere is often coupled with a blocking formation. The importance of transient eddy forcing by synoptic disturbances to maintain low-frequency planetary waves has been a focus of interest in recent years. Many blocking episodes appear to be related to the transient eddy forcing. However, there are exceptions where enhanced transient eddy forcing failed to create blockings, or blockings occurred without significant transient eddy forcing (see Wiin-Nielsen 1986; Shilling 1986). There is as yet no universally accepted theory of blockings, and the causal relationship of the amplification and blockings remains unclear.

Amplification of planetary waves in the troposphere implies an increase of the wave energy. If total atmospheric energy is partitioned in three energy boxes (zonal, planetary waves, and synoptic to short waves), there are only three paths by which planetary wave energy can increase (see Saltzman 1957): (i) down-scale energy cascade from zonal to planetary wave energy; (ii) up-scale energy cascade from synoptic and short wave to planetary wave energy; (iii) energy supply from external forcing. Every theory describes a unique energy flow to excite the planetary waves, and we can classify a number of theories using these three paths.

According to Shilling's (1986) comprehensive analysis, blocking occurrence shows highest coherence with baroclinic instability of planetary waves: down-scale energy cascade. However, there are cases in which barotropic instability plays an important role, and cases in which the up-scale energy cascade from synoptic disturbances dominates the other energy supplies. This suggests that the blocking system is excited by various energy sources from case to case, but reveals the same characteristic structures and behaviors.

In the light of Shilling's puzzling results, Tanaka and Kung (1989) discussed a possibility that blockings can be understood as atmospheric eigenmodes excited by different energy sources from case to case. We intuitively understand the common persistent features as a low-frequency eigenmode. The characteristic structure may be understood such that the eigenvector has the dipole configuration. The eigenmode may be excited by various energy or vorticity supplies because it is a free mode. The positive and negative anomalies should have similar structures (see Dole 1986). We examined eigenmodes of low-frequency, unstable planetary waves in the zonally varying basic state, using spectral primitive equations on a sphere. Two different types of slow-moving Charney modes are found in planetary waves, showing different meridional structures. One of the Charney modes, M_1, is stationary at a preferred geographical location, indicating nearly barotropic structure. It resembles so-called Ω blockings in the atmosphere. The other Charney mode, M_2 indicates a dipole structure in the zonally varying basic state. The structures and behaviors of the dipole Charney mode markedly resemble dipole blockings in the atmosphere. We proposed that dipole Charney modes of wavenumber 1, which is modulated by the steady wavenumber 2, is responsible for large-scale dipole blocking, supported, for example, by the up-scale energy cascade from synoptic disturbances. Yet, it is necessary to confirm the hypothesis using a fully nonlinear time-dependent model, because our previous results are based on a linear model under a restriction of small amplitudes.

The purpose of this study is to simulate the amplification of low-frequency planetary waves and concurrent blocking formations as realistically as possible, using a fully nonlinear spectral primitive equation model which is as simple as possible. The hypothesis of the blocking formation due to the up-scale energy cascade from synoptic disturbances under the persistent wavenumber 2 is examined. The energy flows among different waves during blocking events are investigated.

2. A description of the spectral primitive equation model

A system of primitive equations in a spherical coordinate of longitude λ, latitude θ, normalized pressure $\sigma = p/p_s$, and normalized time $\tau = 2\Omega t$ may be reduced to three prognostic equations of horizontal motions and thermodynamics. The three dependent variables are horizontal wind speeds, $V = (u, v)$, and geopotential deviation ϕ from the global mean reference state. Using a three-dimensional spectral representation, these equations may be written as:

$$\frac{dw_i}{d\tau} + i\sigma_i w_i = -i \sum_{j=1}^{M} \sum_{k=1}^{M} r_{ijk} w_j w_k + f_i, \quad i = 1, 2, ..., M, \quad (1)$$

where w_i and f_i are the Fourier expansion coefficients of dependent variables and diabatic processes, σ_i are Laplace's tidal frequencies, r_{ijk} are interaction coefficients, and M is the total number of the series expansion for the 3-D atmospheric variables. Refer to Tanaka and Sun (1990) for the details. Any choice of expansion basis functions will result in the representation of (1) after a proper diagonalization of the linear terms. The resulting expansion basis functions will consist of vertical normal modes and Hough harmonics. The vertical normal modes comprise barotropic and baroclinic components.

We demonstrated that observed features of blockings can be represented sufficiently by their barotropic components. Based on this observed fact, we collect only the barotropic components of the expansion coefficients. The rest of the baroclinic-barotropic interaction terms and diabatic terms are combined in a single term designated as s_i, which describes the formal source-sink term of the barotropic model:

$$\frac{dw_i}{d\tau} + i\sigma_i w_i = -i \sum_{j=1}^{N} \sum_{k=1}^{N} r_{ijk} w_j w_k + s_i, \quad i = 1, 2, ..., N, \quad (2)$$

where

$$s_i = (BI)_i + (DF)_i + (TF)_i + (ZS)_i. \quad (3)$$

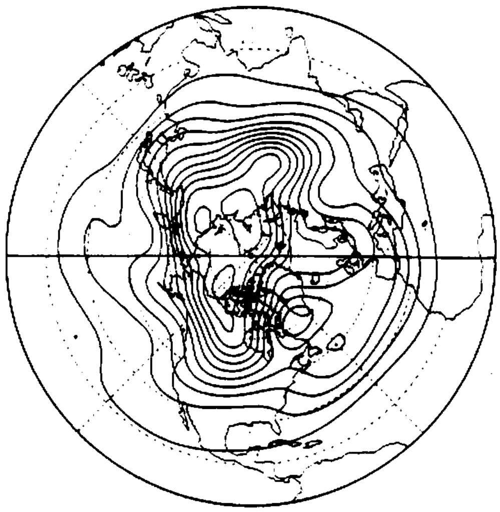
Here, N is the total number of the series expansion for the barotropic model, and $(BI)_i$, $(DF)_i$, $(TF)_i$, $(ZS)_i$ are respectively the formal source-sink terms derived from baroclinic instability, diffusion, topographic forcing, and zonal surface stress. Topography is considered only at the wavenumber 2. Refer to Tanaka (1991) for the details. Given the formal source-sink term s_i, the nonlinear equation (2) becomes a closed system of the prognostic equation.

It is important to notice that the barotropic component of the diabatic heating term becomes zero under a minor assumption, since the heating may be assumed to be zero under the ground. Every heat-related energy source in the atmosphere goes to the baroclinic components, and the energy is then transformed into the barotropic component through the baroclinic-barotropic interaction. This is one of the major attractions of constructing the barotropic primitive equation model from the 3-D spectral model. The complicated heating fields produced by numerous physical processes are concentrated to the single concept of the baroclinic-barotropic interaction.

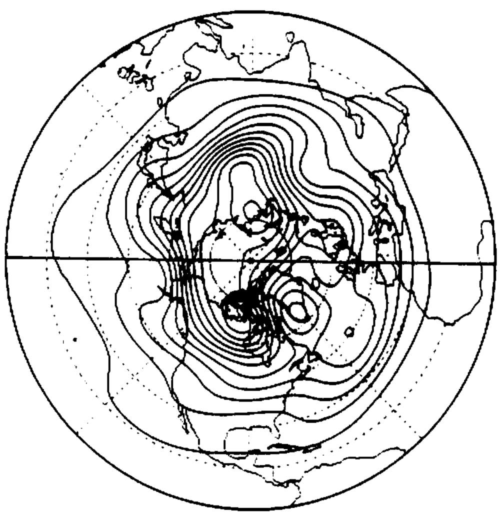
3. Results of the simulation

Figure 1 illustrates daily barotropic geopotential fields during days 54-60. Illustrated is the period when the wavenumber 1 amplifies persistently, indicating a meridional dipole structure. The barotropic geopotential field roughly corresponds to the 500 mb height field. A

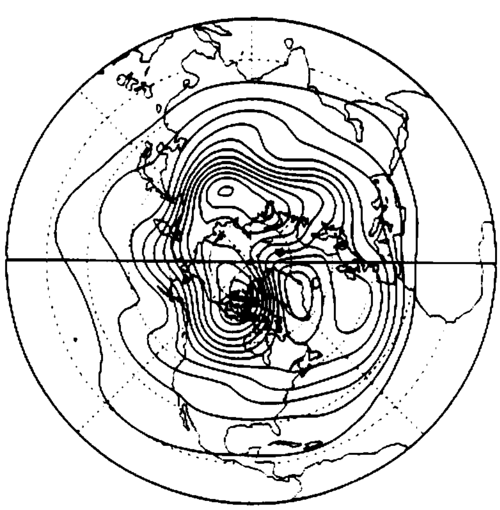
Day 54



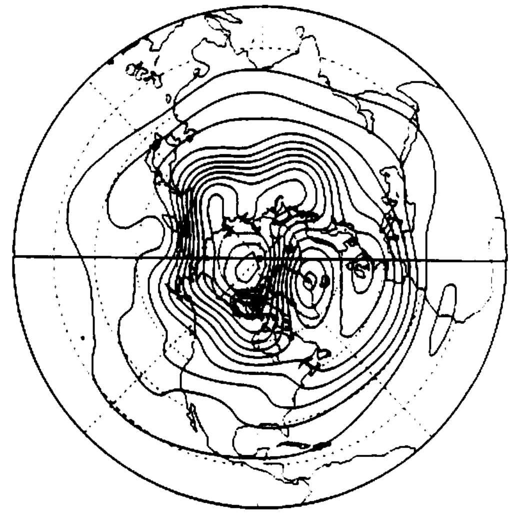
Day 55



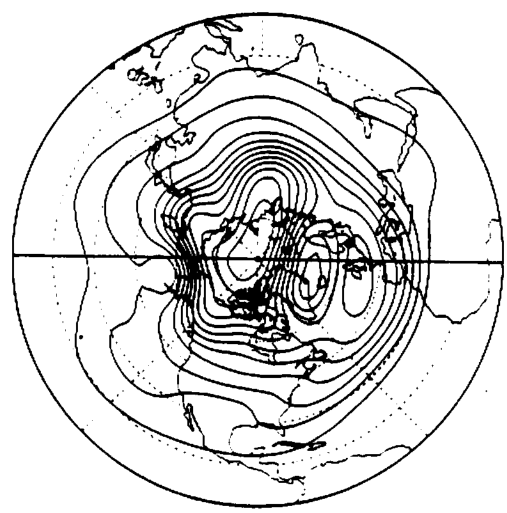
Day 56



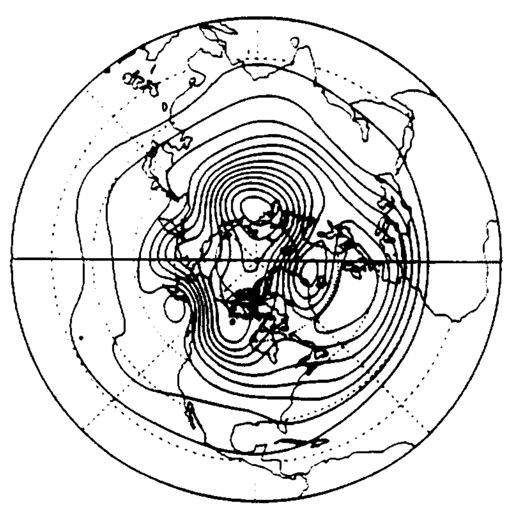
Day 57



Day 58



Day 59



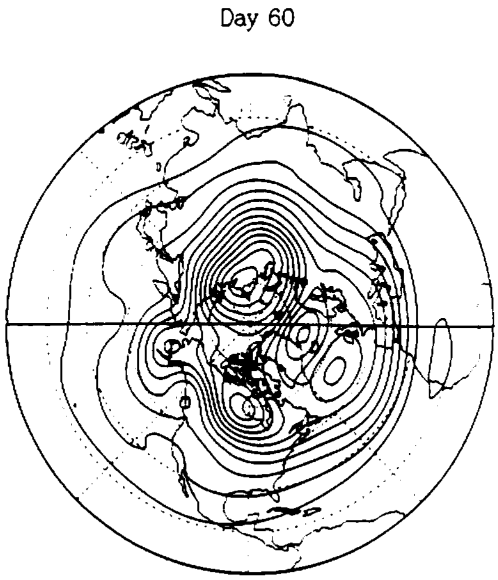


Fig. 1. Daily barotropic geopotential fields during days 54-60 when a blocking occurred in the model atmosphere. The contour interval is 100 m.

blocking system developed near 45°W, indicating a progression. A dipole blocking emerged with its high pressure center at 60°N. The geopotential height of the high-pressure cell is about 300 m higher than surrounding area. Clearly, the dipole wavenumber 1 is superimposed on the amplified wavenumber 2 with its troughs along 90°E and 90°W.

The results described above reasonably resemble observed blocking evolution. A sharp transition from zonal to meridional flow is clear at the up-stream of the blocking system. The persistency of the system is more than two weeks. Our simple nonlinear barotropic model seems to capture the essential mechanism of the blocking system. It is concluded, at least, that the blocking can be simulated using a barotropic model with four physical processes of diffusion, topographic forcing, baroclinic instability, and zonal surface stress.

The mean energy spectrum and energy transformations for days 30-200 are summarized in the Table. The symbols designate total energy E_n, nonlinear interaction NL_n, diffusion DF_n, topographic forcing TF_n, baroclinic instability BI_n, and zonal surface stress ZS_n. The units are $10^3\,Jm^{-2}$ for energy and $10^{-3}\,Wm^{-2}$ for energy transformations. The important role of the nonlinear interaction is evident in energy transfer from the source to the sink in this model.

n	E_n	NL_n	DF_n	TF_n	BI_n	ZS_n
0	1136	258	-11	-44	0	-203
1	131	25	-28	0	11	0
2	204	-185	-54	207	32	0
3	76	4	-35	3	31	0
4	72	-9	-39	9	39	0
5	56	-15	-38	-2	58	0
6	50	-83	-48	-2	138	0

The amplification of the meridional dipole structure of $n=1$ is important for the development of the large-scale dipole blocking. The amplification of $n=2$ is also required. We demonstrated that a model run without topography fails to simulate the large-scale blocking because the planetary waves were not amplified. To analyze the cause of the amplification of $n=1$ and 2, an energy budget analysis is conducted for low-frequency variations during the blocking period. The

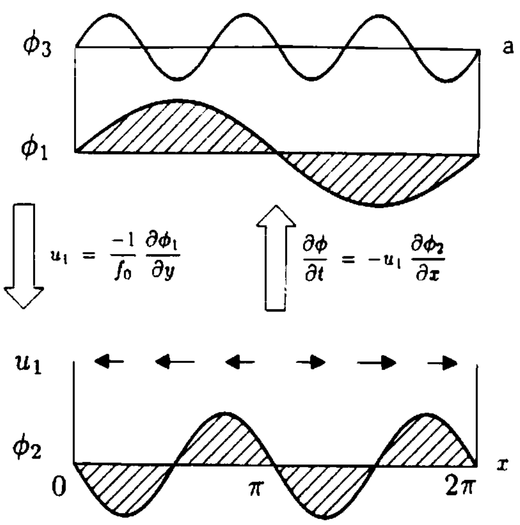


Fig. 2. A schematic diagram of the up-scale self-interaction of wavenumber 1 under the environment of steady wavenumber 2. Suppose that a small disturbance of wavenumber 1, u_1, interacts with steady wavenumber 2, ϕ_2. The nonlinear interaction between $n=1$ and 2 causes amplification of ϕ_1 and ϕ_3. Then the ϕ_1 causes the amplification of u_1 by a geostrophic balance requirement. This closed loop of positive feedback between $n=1$ and 2 amplifies the wavenumber 1 drawing energy of the wavenumber 2.

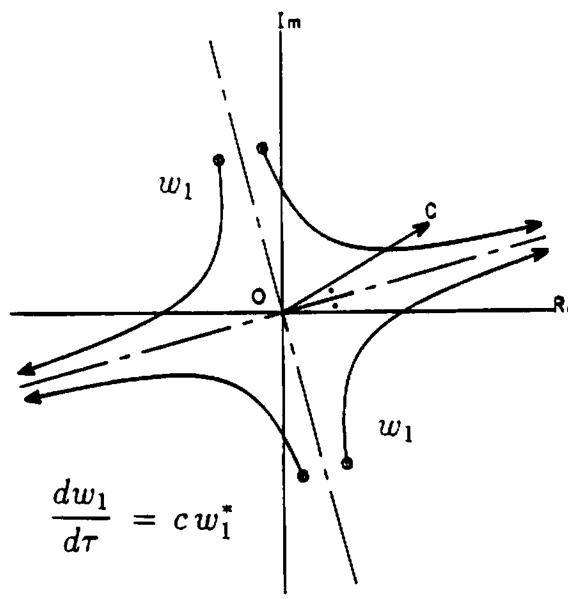


Fig. 3. A schematic diagram of the trajectory of w_1 in multi-dimensional phase space. Suppose that triad wave-wave interactions among $n=0$, 1, and 2 dominate the rest of wave-wave interactions in (1) and that $n=0$ and 2 are in quasi-equilibrium. A condition for dominant up-scale interaction with $n=2$ (i.e., vanishing down-scale interaction) is satisfied when a low-frequency unstable mode of $n=1$ becomes stationary under a weak dissipation. The growing direction describes the unstable manifold and the decaying direction, the stable manifold. The trajectory tend to approach the unstable manifold and grow exponentially. This implies that the low-frequency Charney mode of $n=1$ is captured by $n=2$.

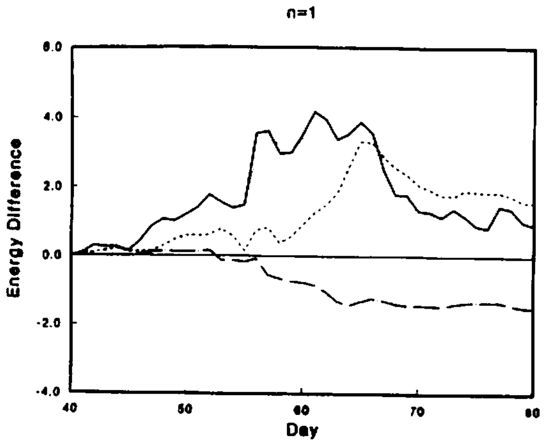


Fig. 4. Time evolution of nonlinear interactions for $n=1$, divided in contributions from down-scale self-interactions with $n=0$, i.e., (1,0,1) for chain-dot line; up-scale self-interactions with $n=2$, i.e., (1,-1,2) for dashed line; and the rest of up-scale triad interactions of (1,-2,3), (1,-3,4), (1,-4,5), and (1,-5,6) for dotted line. The solid line describes the sum of these three lines. Units are $10^5 Jm^{-2}$.

results show that the low-frequency variation of $n=1$ is almost completely explained by the nonlinear wave-wave interactions, and other physical processes show secondary importance. The energy variation for $n=2$ is caused by two competing processes of topographic forcing, and the nonlinear scattering. During the amplification of $n=1$, the wavenumber 2 draws energy from topographic forcing and feeds the energy for other waves through the nonlinear interactions. The time variations of wavenumbers 1 and 2 are negatively correlated.

A possible mechanism of the up-scale self-interaction of wavenumber 1 under persistent wavenumber 2 is illustrated in Figs. 2 and 3 with a simple model. Refer to the descriptions of the figure legends. Robinson (1985) discussed a growth of $n=1$ as an expense of energy of $n=2$ as well as $n=0$. It is enticing to support a topographic origin for the amplification of wavenumber 1. However, we find by a detailed analysis that this is not the case.

The important problem is to show from where the energy of $n=1$ is transformed. Every triad wave-wave interaction must satisfy a wavenumber rule $n=n'+n''$. For $n=1$ the triad interactions (n,n',n'') can be classified in three types: down-scale self-interactions with $n=0$, i.e., (1,0,1); up-scale self-interactions with $n=2$, i.e., (1,-1,2); and the rest of up-scale triad interactions of (1,-2,3), (1,-3,4), (1,-4,5), and (1,-5,6). Any up-scale interaction toward smaller wavenumber involves a triad combination with a negative wavenumber. The negative wavenumber appears as a complex conjugate term in Eq. (2), and its mathematical role in the system is quite different from that of a positive wavenumber (see Fig. 3).

With this classification, the nonlinear wave-wave interactions of $n=1$ are divided in contributions from these three types of interactions. Figure 4 illustrates the result of transient energy interactions integrated with respect to time. The chain-dot line (1,0,1) describes zonal-wave interactions of ordinary barotropic conversion. The negative indicates an enhanced acceleration of the zonal jet. The dashed line (1,-1,2) describes the interaction with $n=2$ where the topographic forcing exists. The result shows an increased energy supply at the end of the blocking episode. The dotted line describes the energy supply from synoptic disturbances of $n=3$-6. There is an increased energy supply at the beginning and mature stages of the blocking episode. The solid line describes the sum of these three lines, which coincides with the energy variation of $n=1$. The results show that the important energy supply into $n=1$ comes from synoptic disturbances through the wave-wave interactions.

It is shown by Shepherd (1987) and Tanaka and Sun (1990) that the synoptic disturbances can be regarded as inhomogeneous turbulence with specific power laws in the meridional energy spectrum. Hence, present result implies that the blocking is created out of turbulence.

4. Concluding remarks

In this study, we carried out nonlinear numerical simulations of amplification of low-frequency planetary waves and concurrent blocking formations. Analyzing energetics of blocking formations in the model, we confirmed an amplification of the meridional dipole mode by means of the up-scale energy cascade from synoptic disturbances. We find that the persistent wavenumber 2 plays a catalytic role in drawing synoptic wave energy and feeding wavenumber 1 and that without it a realistic blocking was not simulated.

The results of our numerical simulation are summarized as follows:

(i) the wavenumber 2 is amplified by the prescribed topographic forcing;

(ii) synoptic disturbances are excited by parameterized baroclinic instability;

(iii) under the quasi-stationary wavenumber 2, a meridional dipole mode of wavenumber 1 is amplified by the up-scale energy cascade from synoptic disturbances;

(iv) basic futures of the blocking are created by the superposition of the amplified wavenumbers 1 and 2, and the synoptic disturbances contribute to the sharp diffluent structure of the zonal jet.

Recent studies of a critical layer solution for breaking Rossby waves show a blocking-like solution (e.g., McIntyre and Palmer 1985). The solution provides us a clear insight of convection and turbulence. We note however that the critical layer solution is associated with a down-scale energy cascade, whereas the blocking is mostly related with the up-scale cascade as demonstrated by this study. Based on this fact, it is inferred that turbulence creates disorder (e.g., chaos) when the energy cascades down scale, whereas it creates order (e.g., zonal jets and blockings) when the energy cascades up scale.

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References

Dole, R. M., 1986: The life cycle of persistent anomalies and blocking over the North Pacific. *Advances in Geophysics*, **29**, 31-69.

McIntyre, M. E., and T. N. Palmer, 1985: A note on the general concept of wave breaking for Rossby and gravity waves. *Pure Appl. Geophys.*, **123**, 964-975.

Robinson, W. A., 1985: A model of the wave 1-wave 2 vacillation in the winter stratosphere. *J. Atmos. Sci.*, **42**, 2289-2304.

Saltzman, B., 1957: Equations governing the energetics of the larger scales of atmospheric turbulence in the domain of wave number. *J. Meteor.*, **14**, 513-523.

Shepherd, T. G., 1987: A spectral view of nonlinear fluxes and stationary-transient interaction in the atmosphere. *J. Atmos. Sci.*, **44**, 1166-1178.

Shilling, H.-D., 1986: On atmospheric blocking types and blocking numbers.— Anomalous Atmospheric Flows and Blocking, Advances in Geophysics, 29, (eds. R. Benzi, B. Saltzman, and A. C. Wiin-Nielsen). Academic Press, 71-99.

Tanaka, H. L., and E. C. Kung, 1989: A study of low-frequency unstable planetary waves in realistic zonal and zonally varying basic states. *Tellus*, **41A**, 179-199.

Tanaka, H. L., and S. Sun, 1990: A study of baroclinic energy source for large-scale atmospheric normal modes. *J. Atmos. Sci.*, **47**, 2674-2695.

Tanaka, H. L., 1991: A numerical simulation of amplification fo low-frequency planetary waves and blocking formations by the up-scale energy cascade. *Mon. Wea. Rev.*, **119**, (in press).

Wiin-Nielsen, A., 1986: Global scale circulations.— A review.— Anomalous Atmospheric Flows and Blocking, Advances in Geophysics, 29, (eds. R. Benzi, B. Saltzman, and A. C. Wiin-Nielsen). Academic Press, 3-27.