Spectral Energetics Analysis of the General Circulation of the Atmosphere in the Vertical Wavenumber Domain

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1. INTRODUCTION

The atmospheric energetics has been investigated since the atmospheric energy flow was discussed by Lorenz (1955) using the concept of the available potential energy. Lorenz (1955) studied the energetics of the atmospheric general circulation with dividing the atmospheric data into zonal and eddy components. Saltzman (1957) expanded the energy equations into the zonal wavenumber domain and showed that the kinetic energy of the cyclone-scale waves is transformed into both the planetary waves and the short waves in term of nonlinear wave-wave interactions. Since Kasahara and Puri (1981) obtained orthonormal eigensolutions to the vertical structure equation, it became possible to expand the atmospheric data into the three-dimensional harmonics of the eigensolutions.

Tanaka and Kung (1988) studied the atmospheric energy spectrum and interactions expanding the atmospheric data to the three-dimensional normal mode functions. The vertical structure functions used by them were obtained by solving the vertical structure equation with a finite difference method. The numerical vertical structure functions have quite large aliasing for higher order vertical modes indicating largest amplitudes near the sea level despite that the analytical solutions indicate the largest amplitudes always in the upper atmospehre (see Sasaki and Chang 1985).

The barotropic-baroclinic interactions have been studied by many researchers (Wiin-Nielsen 1962; Smagorinsky 1963). Wiin-Nielsen (1962) investigated the kinetic energy interactions between the vertical shear flow and the vertical mean flow. According to their analysis, the energy conversion between shear flow and mean flow is about 30 percent of the conversion between the available potential energy and the shear flow kinetic energy.

Terasaki and Tanaka (2007) studied the three-dimensional atmospheric energetics using the analytical vertical structure functions. In their study, it was found that the energy spectrum indicates a clear peak in the middle vertical modes, and the spectrum decreases monotonically at the higher order vertical modes. The energy interactions for lower order vertical modes are consistent with that by Tanaka and Kung (1988). However, it is found from the analysis of the energy interactions that there is another energy source region in the higher order vertical modes in the zonal field.

2. GOVERNING EQUATION AND DATA

2.1 Primitive Equation

The governing equations used in this study are the primitive equations, consisting of equation of motions, thermodynamic equation, hydrostatic equation, equation of state, and law of mass conservation. A system of primitive equations is defined on a spherical coordinate of longitude λ, latitude θ, nondimensional pressure $\sigma = p/p_s$ ($p_s = 1000$ hPa), and time t. Where p_s is constant surface pressure.

2.2 Vertical Structure Functions

The basis function for the vertical direction is derived by solving the vertical structure equation as follows (Terasaki and Tanaka 2007):

$$G_0(\sigma) = C_{10}\sigma^{b_{10}} + C_{20}\sigma^{b_{20}}, \quad (1)$$

$$G_m(\sigma) = \sigma^{-\frac{1}{2}}\{C_{1m}\cos(\mu_m \ln\sigma) + C_{2m}\sin(\mu_m \ln\sigma)\}, \quad (2)$$

$$b_{1m} = -\frac{1}{2} + \mu_m,\ b_{2m} = -\frac{1}{2} - \mu_m,$$
$$\mu_m = \sqrt{|\ \frac{1}{4} - \lambda_m\ |}, \quad (3)$$

where the eigenvalues λ_m are obtained by solving the eigenvalue problem of vertical structure equation. C_{1m} and C_{2m} are obtained from the boundary conditions at the surface and the top of the atmosphere, and normalized as $C_{1m}^2 + C_{2m}^2 = 1$. The vertical structure functions have a orthogonality, so the orthonornal basis functions can be obtained by dividing with the norm of the vertical structure functions.

Figure 1 shows the vertical profiles of the analytical vertical structure functions for vertical modes $m = 0$ to 5. The envelope function of $\sigma^{-1/2}$ is superimposed in the figure. The analytical vertical structure functions for baroclinic modes are represented by trigonometric functions multiplied by the envelope function, so the profiles have larger amplitudes at the upper atmosphere. The subscript m of the vertical structure functions represents only the vertical mode number and doesn't indicate vertical scale. But by using μ_m in (2), we can express a nondimensional vertical wavenumber in the same manner as the Fourier expansion. We use both vertical indices in this study.

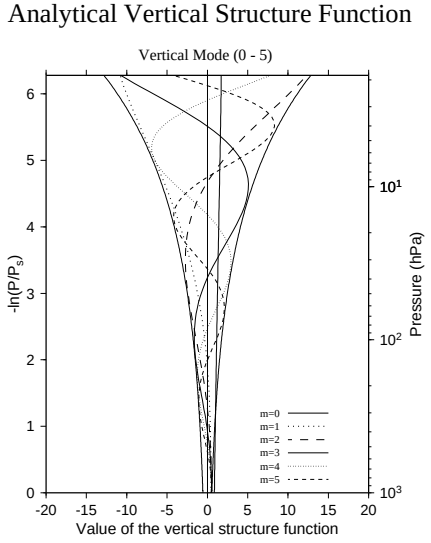

Figure 1: The vertical profiles of the analytical vertical structure functions for $m = 0 - 5$.

2.3 Global Energy Budget Equations

In order to obtain the energy budget equations, we summarize the kinetic energy and available potential energy equations as:

$$\frac{\partial K_m}{\partial t} = -M(m) + L(m) + C(m) - D(m), \quad (4)$$

$$\frac{\partial K_0}{\partial t} = \sum_{m=1}^{M} M(m) + C(0) - D(0), \quad (5)$$

$$\frac{\partial P_m}{\partial t} = R(m) + S(m) - C(m) + G(m), \quad (6)$$

$$\frac{\partial P_0}{\partial t} = -\sum_{m=1}^{M} R(m) - C(0) + G(0). \quad (7)$$

Eqs. (4) - (7) are the energy budget equations for the baroclinic kinetic energy, barotropic kinetic energy, baroclinic available potential energy, and barotropic available potential energy, respectively. The atmospheric energy flows in the vertical spectral domain can be examined by calculating these terms.

The data used in this study are four-times daily (00, 06, 12, and 18 UTC) JRA-25 (Japanese Re-Analysis 25 years) (Onogi et al. 2007) and JCDAS (JMA Climate Data Assimilation System) from 1979 to 2007. The data contain meteorological variables of horizontal wind u, v, vertical p-velocity, temperature, and geopotential ϕ, defined at every 2.5° longitude by 2.5° latitude grid points over 23 mandatory vertical levels from 1000 to 0.4 hPa. The data are interpolated on the 46 Gaussian vertical levels in the log (p/p_s) coordinate by the cubic spline method.

3. RESULTS

Figure 2 illustrates kinetic energy and available potential energy cycle between the barotropic and baroclinic components for the Northern Hemisphere, evaluated for 27 years using the JRA-25 and JCDAS. The energeric terms of the baroclinic modes are derived by summing up each term of all baroclinic modes. The baroclinic-baroclinic interactions of kinetic and available potential energies vanishes when they are summed up in all baroclinic modes. A large amount of available potential energy is included in the baroclinic mode. The energy of

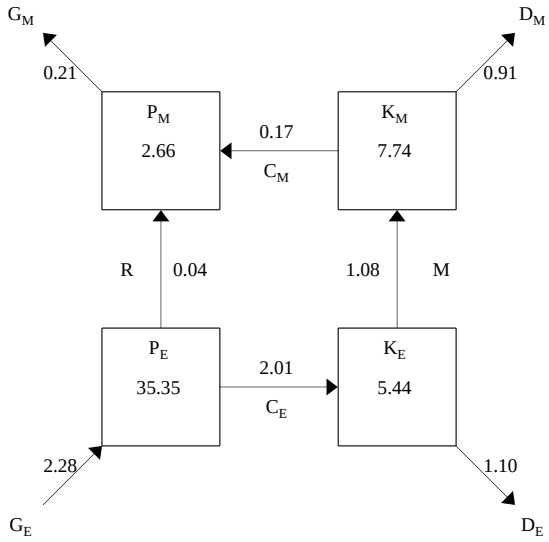

Figure 2: The kinetic and available potential energy cycle for the barotropic and baroclinic components of the Northern Hemispheric atmosphere. The units of the energy are $10^5 \mathrm{J/m^2}$, and those of the interactions term are $\mathrm{W/m^2}$.

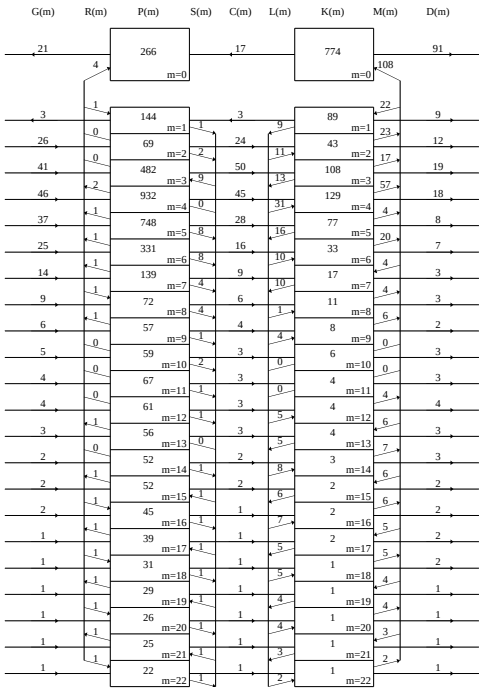

Figure 3: The energy flow diagram of the atmospheric general circulation in the vertical spectral domain. The units of the energy are $10^3 \mathrm{J/m^2}$, and those of the interactions term are $10^{-2} \mathrm{W/m^2}$.

the atmospheric general circulation is injected as the baroclinic available potential energy by the solar heating. The amount of the energy injection is 2.28 $\mathrm{W/m^2}$. The baroclinic conversion, which is the energy conversion from available potential energy to kinetic energy by the baroclinic instability, is 2.01 $\mathrm{W/m^2}$. A part of this baroclinic kinetic energy is dissipated by the viscosity or friction, and the amount of the dissipation is 1.10 $\mathrm{W/m^2}$. The residual baroclinic kinetic energy is transformed to the barotropic motion. Finally, the barotropic kinetic energy is dissipated by the viscosity or friction.

Figure 3 shows the kinetic energy and available potential energy flows in the vertical wavenumber domain. The similar analysis in the zonal wavenumeber domain, in which the Fourier expansion is used for basis functions, is performed by Saltzman (1957). It is found in this study that the generation of the baroclinic available potential energy is widely distributed to the higher order vertical modes, while the maximum injection is seen at the lower order vertical modes around $m = 4$. The barotropic available potential energy actually should be zero, if the barotropic mode strictly means vertical mean. But the vertical structure function of vertical mode $m = 0$ doesn't have a constant value, so the available potential energy of the barotropic mode has a nonzero value. It is found that the interactions of the available potential energy between baroclinic-baroclinic and barotropic-baroclinic are very small compared to that of the kinetic energy. The baroclinic available potential energy is directly converted to the same scale of the kinetic energy without interacting among them. Most of the baroclinic conversions have positive values except for the barotropic mode ($m = 0$) and one of the baroclinic modes ($m = 1$). It is found that the baroclinic kinetic energy interacts within baroclinic modes, and then they are transformed to the barotropic mode. The energy interactions in higher order vertical modes have a zigzag distribution. This is caused by the comutational error that the vertical structure function in the upper atmosphere has a large amplitude.

4. CONCLUSION

In this study, a new method for the spectral analysis of the atmospheric general circulation in the vertical wavenumber domain is introduced, using the analytical vertical structure functions as a basis function for the vertical direction. The analytical vertical structure functions can be obtained by assuming the static stability parameter γ to be a constant value. The vertical expansion is applied to a system of the primitive equations in consideration of the proper boundary conditions, and the kinetic energy and available potential energy equations are derived. Using this analysis method, we can examine the interactions of kinetic and available potential energies within baroclinic modes.

According to the result of the analysis in dividing the atmospheric data into vertical mean (barotropic) and its shear (baroclinic), we obtain the energy cycle of the atmospheric general circulation. This result is consistent with previous studies.

According to the result of the analysis in the vertical wavenumber domain, it is found that the baroclinic kinetic energy interacts within baroclinic modes, and then they are transformed to the barotropic mode. The interactions for available potential energy are very small compared to that for the kinetic energy. Further analysis must be desired about the energy interactions within the baroclinic components, and its spatial distribution to analyze the various atmospheric phenomena.

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