Global Energetics Analysis Expansion into Vertical Structure Function

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1. INTRODUCTION

Since an atmospheric energy flow was discussed by Lorenz (1955) using the concept of available potential energy, the energetical role of the atmospheric eddies has been extensively investigated. Saltzman (1957) expanded the energy equations into the zonal wavenumber domain and showed that kinetic energy of the cyclone-scale waves is transformed to both planetary waves and short waves in terms of nonlinear wave-wave interactions.

The vertical energy spectrum was studied using vertical structure functions by Kasahara and Puri (1981). As discussed by Tanaka (1985), the atmospheric data were expanded using the numerical solutions of the vertical structure equation derived by Kasahara and Puri (1981). But the numerical solutions have quite large aliasing in higher order vertical modes. Since the vertical energy spectrum has been calculated using vertical structure functions by Kasahara and Puri (1981), the vertical energy spectrum has large aliasing in higher order vertical modes.

The analytical solutions of the vertical structure equation were investigated by Fukutomi(1994). The analytical solution can be obtained by assuming the static stability as a constant. When the vertical structure functions without the aliasing are used, we can calculate the non-aliased vertical energy spectrum.

The purpose of this study is to analyze the vertical energy spectrum of the general circulation by using the analytical solutions of the vertical structure equation which have no aliasing. Since the vertical energy spectrum is evaluated at all latitudes, it may be interesting to compare the characteristics of the energy spectrum. For this reason, we investigate, in the second, the difference of the energy spectra in three regions of the polar region, the mid-latitudes, and the tropics.

Table 1: The list of the equivalent heights in meter for the vertical modes $m = 0$ to $m = 10$.

m	equivalent height (h_m)
0	9726
1	1861
2	797
3	410
4	244
5	161
6	113
7	84
8	64
9	51
10	41

2. EQUATION AND DATA

When the static stability defined in Tanaka

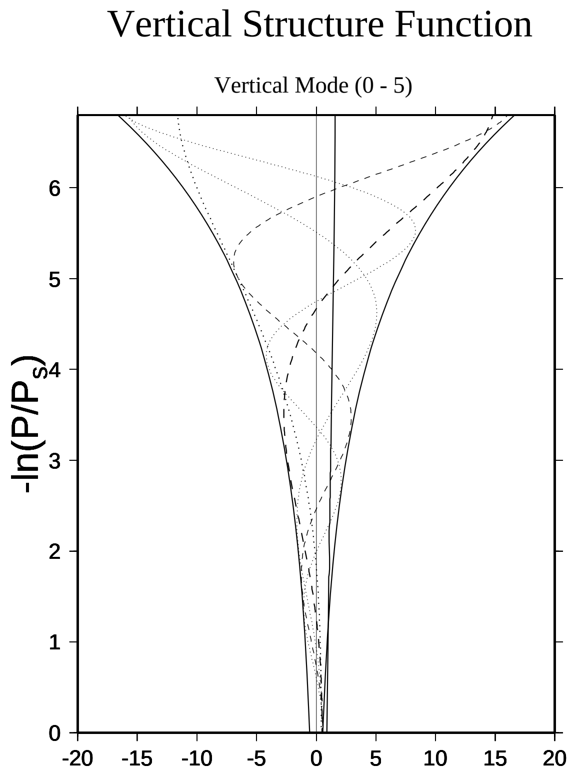

Figure 1: Analytical vertical structure functions for vertical modes m from 0 to 5.

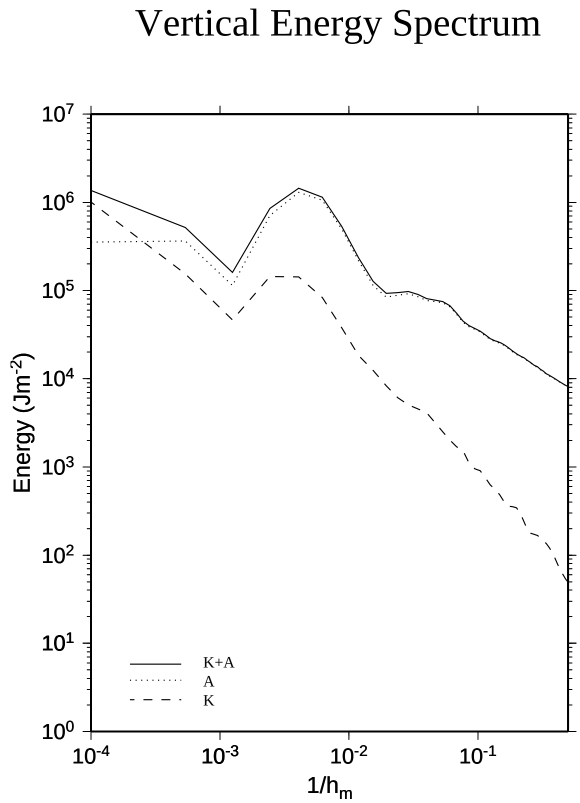

Figure 2: The distributions of the global mean vertical energy spectrum as a function of $1/h_m$. Solid, dotted, and dashed lines denote total energy, available potential energy, and kinetic energy.

(1985) is assumed to be a constant, the vertical structure equation in terms of $\sigma(= p/p_s)$ coordinate is reduced to so-called Euler equation, where p_s is the surface pressure. The analytical solutions of the vertical structure equation is described as follows:

$$G_0(\sigma) = C_1\sigma^{r_1} + C_2\sigma^{r_2}, \quad (1)$$

$$G_m(\sigma) = \sigma^{-\frac{1}{2}}\{C_1\cos(\mu\ln\sigma) + C_2\sin(\mu\ln\sigma)\}, \quad (2)$$

where r_1, r_2, and μ can be obtained from the eigenvalue of the vertical structure equation, and C_1 and C_2 are the constants determined by boundary conditions. The solutions expressed by (1) and (2) represent barotropic (external) and baroclinic (internal) modes, respectively, and the structures for the vertical modes m = 0 to 5 are plotted in Fig. 1. The structure of the barotropic mode m=0 is nearly constant at all levels. The values of the exponent are r_1=-0.1. That of baroclinic modes are confined in the curve of $\sigma^{-0.5}$ multiplied by sinusoidal curves of vertical wavenumber μ in log-σ coordinate.

In this study, in order to investigate the vertical energy spectrum, we expand the state variables of the atmosphere $\mathbf{U} = (u, v, \phi')^T$ using the vertical structure functions, where u, v, ϕ' represent zonal and meridional components of wind and geopotential deviation from the global mean reference state, respectively:

$$\mathbf{U}(\lambda,\theta,\sigma,t) = \sum_{m=0}^{\infty} \mathbf{U}_m(\lambda,\theta,t)G_m(\sigma), \quad (3)$$

where $G_m(\sigma)$ is the vertical structure function, and $\mathbf{U}_m(\lambda,\theta,t)$ is the vertical expansion coefficient for the vertical index m in the subscripts. The vertical expansion coefficient may be calculated by the vertical integration:

$$\mathbf{U}_m(\lambda,\theta,t) = \int_1^0 \mathbf{U}(\lambda,\theta,\sigma,t)\mathbf{X}_m G_m(\sigma)d\sigma, \quad (4)$$

where $\mathbf{X}_m$ is a scaling matrix represented by

$$\mathbf{X}_m = diag(\sqrt{gh_m}, \sqrt{gh_m}, gh_m). \quad (5)$$

The energy of the atmosphere can be com-

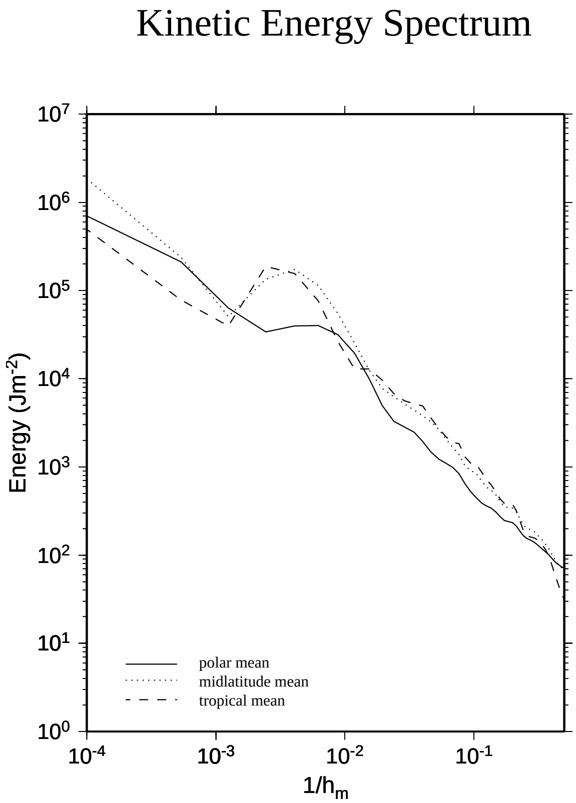

Figure 3: The kinetic energy spectrum in the vertical mode domain. Solid, dotted, and dashed lines are for polar region, mid-latitudes, and the tropics.

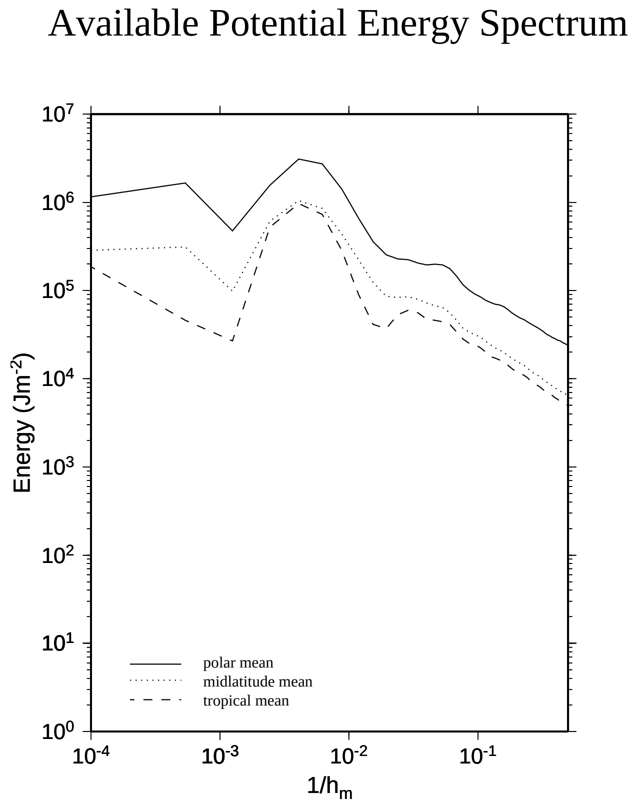

Figure 4: As in Fig. 3, but for available potential energy spectrum in the vertical mode domain.

puted for each vertical mode:

$$E_m = \frac{1}{2} p_s h_m |\mathbf{U}_m|^2, \tag{6}$$

where h_m is the equivalent height evaluated as the inverse of the eigenvalues of the vertical structure equation. The equivalent height from $m = 0$ to $m = 10$ is listed in Table 1. The equivalent height of the barotropic mode is 9726 m. The energy spectrum E_m is plotted as a function of the inverse of the equivalent height.

The data used in this study are four-times daily JRA-25 for January 1978. The data contain horizontal winds (u, v) and geopotential ϕ, defined at every 2.5°longitude by 2.5°latitude grid point over 23 mandatory vertical levels from 1000 to 0.4 hPa.

3. ENERGY SPECTRUM

Figure 2 illustrates the vertical energy spectrum for the global mean as a function of the inverse of h_m. Solid, dotted, and dashed lines denote total energy, available potential energy, and kinetic energy. According to the result, the spectrum shows two energy peaks at the barotropic mode and one of the baroclinic modes. A large amount of kinetic energy is included in the barotropic mode. The available potential energy of the barotropic mode is quite small in contrast to the kinetic energy of barotropic mode. The available potential energy should be zero for m=0, if the vertical structure function were exactly constant with respect to pressure. It is, however, not zero because the structure is represented by (1) which is not constant.

Another peak of kinetic energy is seen at m=4 which is one of the baroclinic modes. As is seen in Fig. 1, the vertical structure function for m=4 indicates large amplitude at about 200 hPa. The tropospheric jet at about 200 hPa may cause the secondary energy peak at m=4. The peak of available potential energy at m=4 is caused by the opposite signs in geopotential at the surface and the tropopause level. In the small equivalent height, both kinetic and available potential energies consistently decrease.

The higher order modes of kinetic energy follow -1.5 power of the inverse of the equivalent height, and those of available potential energy follow -1.0. In the higher order modes, total energy also follows -1.0 power, since the available potential energy is two orders of magnitude more than kinetic energy.

Figure 3 illustrates the kinetic energy distributions over three regions, i.e., the polar region (solid line), the mid-latitudes (dotted line) and the tropics (dashed line). In the barotropic mode, the kinetic energy of the mid-latitudes is the largest among the three regions. At the baroclinic modes around $h_m = 150$ m, the kinetic energies in the tropics and mid-latitudes are similar to the global mean. But that in the polar region is quite different from other two regions and the global mean. The energy peak for the polar region shifts to the smaller equivalent height, and the peak is reduced.

Figure 4 illustrates the available potential energy distribution over three regions as in Fig. 3. The available potential energy is characterized by the temperature deviation from the global mean. The available potential energy for the polar region is the largest among the three regions at any vertical modes, because the temperature of the polar region is much lower than that of the global mean.

4. CONCLUSION

In this study, the vertical energy spectrum of the general circulation is examined using the analytical vertical structure functions as solutions of Euler equation. The vertical energy spectrum can be calculated with higher accuracy compared to that obtained by numerical method, because there is no aliasing in the vertical structure functions.

As a result of analysis, we obtain the vertical energy spectrum over the wide range of the inverse of the equivalent heights in baroclinic modes. The kinetic energy indicates the energy maximum at the barotropic mode. Another peak of the kinetic energy is seen in the baroclinic modes located at about $h_m = 150$ m. This result reflects the structure of the subtropical jet with the peak at the tropopause. As the vertical scale becomes smaller, the kinetic energy level becomes smaller continuously.

The most of the available potential energy of the atmosphere is included in the baroclinic modes at h_m equals to about 150 m. The large amount of the available potential energy at this vertical scale reflects the vertical structure of geopotential with opposite signs at the surface and the tropopause level.

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REFERENCES

Fukutomi, Y., 1994: Application of vertical spectral expansion to problem of atmospheric baroclinic instability. Graduation Thesis, College of Natural Science, University of Tsukuba, 52 pp.

Kasahara and K. Puri, 1981: Spectral representation of three dimensional global data by expansion in normal mode functions. *Mon. Wea. Rev.*, **109**, 37–51.

Lorentz, E. N., 1955: Available potential energy and the mainenance of the general circulation. *Tellus*, **7**, 157–167.

Saltzman, B., 1957: Equations governing the energetics of the larger scales of atmospheric turbulence in the domain of wavenumber. *J. of Meteorology.*, **14**, 513-523.

Tanaka, H.L., 1985: Global energetics analysis by expansion into three dimensional normal mode functions during the FGGE winter. *J. Meteor. Soc. Japan*, **63**, 180–200.