Dynamical Understanding of the Arctic Oscillation as a Singular Eigenmode of the Global Atmosphere

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1. INTRODUCTION

The Arctic Oscillation (AO) is a north-south seesaw of the atmospheric mass between the Arctic region poleward of 60°N and a surrounding zonal ring in the mid-latitudes. The AO is defined as the EOF-1 of the time variation in sea-level pressure p_s, which is dynamically related to the time variation of the barotropic component of the atmosphere as follows:

$$\frac{\partial p_s}{\partial t} \simeq -\int_0^{p_s} \nabla \cdot V dp \simeq -p_s \nabla \cdot V_0 \simeq \frac{p_s}{g h_0} \frac{\partial \phi_0}{\partial t}. \quad (1)$$

Here, the time variation of p_s is caused by the vertical integral of mass flux convergence $\nabla \cdot V$. The vertical integral coincides with the barotropic component of the atmosphere (noted by the subscript 0) which controls the time variation of the barotropic geopotential ϕ_0 in a shallow water system with the depth h_0 and gravity g. Hence, the AO represented by the variation in p_s is dynamically equivalent to the variation in ϕ_0. For this reason, the essential features of the AO are contained in the barotropic component of the atmosphere governed by the 2D fluid mechanics which characterizes the low-frequency variability.

Using a simple barotropic model derived from the 3D normal mode expansion, Tanaka (2003) conducted a numerical simulation of the AO and obtained the same structure as observed in the atmosphere. According to the 50 years time integration of the model, not only the EOF-1 of the AO mode but also the EOF-2 agrees well with the observation which is identified as the Pacific North America (PNA) pattern. The low-frequency variabilities such as the AO and PNA are evidently contained in the internal dynamics of the barotropic component of the atmosphere.

Beside the controversy to recognize the AO as a dynamical mode or a statistical artifact, a dynamical approach has been pursued by Tanaka and Matsueda (2005) by solving a singular mode with the smallest singular value of the linearized dynamical system, which is now referred to as the neutral mode theory (e.g., Kimoto et al. 2001; Watanabe and Jin 2004). Tanaka and Matsueda (2005) identified that the characteristics of the singular mode resembling with the AO are originated from the eigenmode of the dynamical system with nearly zero eigenvalue, i.e., singular eigenmode for the global atmosphere.

It is demonstrated that the singular eigenmode of the dynamical system emerges resonantly as the AO in response to the arbitrary forcing under a moderate linear damping. However, it is still unclear for the relation between the singular eigenmodes and the neutral modes under the weak and strong linear damping, re-

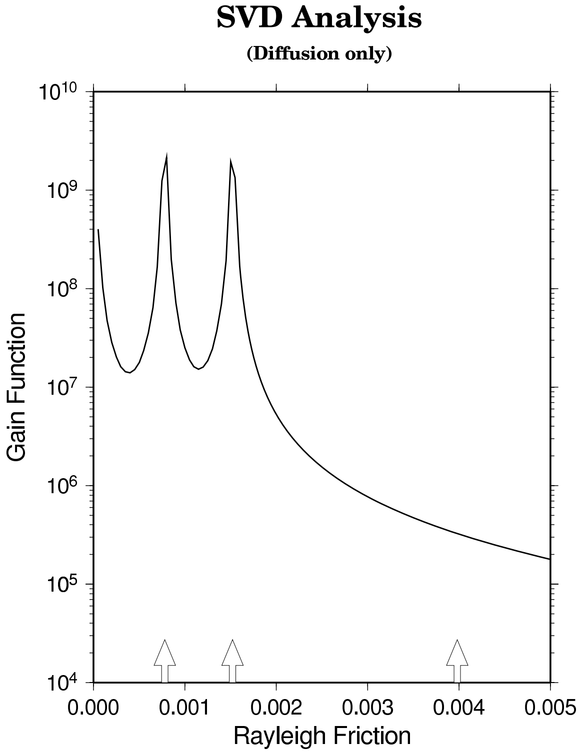

Figure 1: A linear response function of the dynamical system as a function of the Rayleigh friction ν_S. The peaks represent the singular eigenmodes EVP-1, EVP-2, EVP-3, respectively, from the right. Arrows indicate the linear damping where SVD analyses are conducted.

spectively.

The purpose of this study is to examine the relation between the singular eigenmodes and the neutral mode by adding a strong linear damping. We first analyze the linear response curve as a function of the linear damping. Then the structures of the neutral modes for different intensities of the linear damping are examined.

2. METHOD AND DATA

According to Tanaka (2003) the 3D representation of the spectral primitive equations on a sphere may be written as

$$\frac{dw_i}{d\tau} = -i\sigma_i w_i - i\sum_{jk} r_{ijk} w_j w_k + f_i, \qquad (2)$$

where w_i is the complex state variable of the system obtained as the Fourier expansion coefficients of meteorological variables, σ_i is the eigenfrequency of the Laplace's tidal equation, f_i is the expansion coefficient of the external forcing of viscosity and diabatic heating rate, and r_{ijk} is the interaction coefficients for the nonlinear system. Refer to Tanaka and Terasaki (2005) for the computation of σ_i and r_{ijk}.

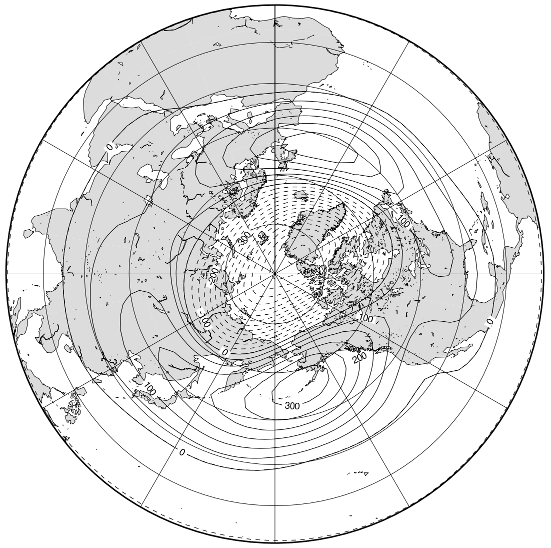

Figure 2: Barotropic height of the singular eigenmode EVP-1 which is easily recognized as the AO pattern.

In this study, the frictional forcing f_i is parameterized by the following hyper diffusion and the linear damping representing Rayleigh friction as in Tanaka and Matsueda (2005):

$$f_i = -k_D \, c_i^{-4} \, w_i - \nu_S \, w_i, \qquad (3)$$

where k_D is a diffusion coefficient, c_i is a phase speed of Rossby modes. The linear damping coefficient ν_S is first set to zero and will be added later to shift the eigenvalues so that the system becomes singular.

The data used in this study are four-times daily NCEP/NCAR reanalysis for 55 years from 1950 to 2004. The spectral coefficients are computed from the reanalysis data and the matrix eigenvalue problem and the singular value decomposition are solved using a standard numerical matrix solver for a given basic state of the DJF mean climate as illustrated in Fig. 1 of Tanaka and Matsueda (2005).

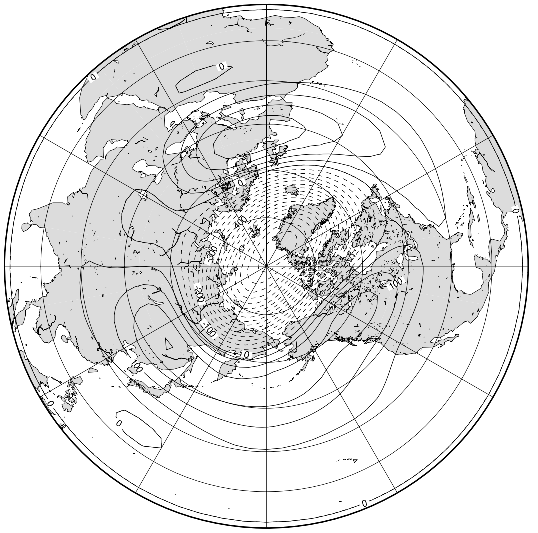

Figure 3: Barotropic height of the SVD-1 for reasonably strong Rayleigh friction at the right-most arrow in Fig. 1.

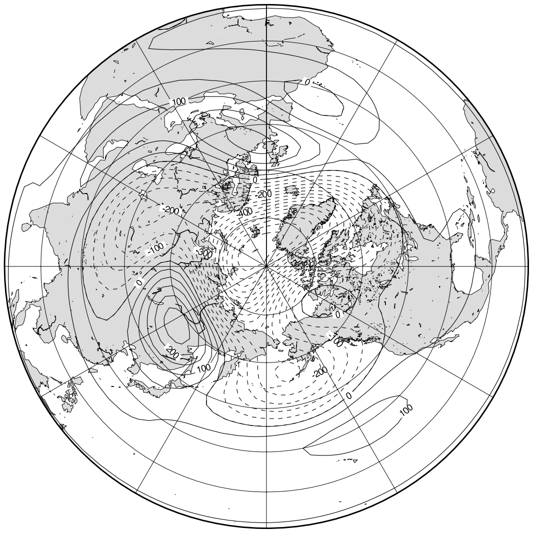

Figure 4: External forcing of the SVD-1 for reasonably strong Rayleigh friction at the right-most arrow in Fig. 1.

3. RESULT

Figure 1 shows the linear response function defined by the inverse of the singular values as a function of the linear damping of Rayleigh friction. As noted by Tanaka and Matsueda (2005), EVP-3 is close to singular so identified as SVD-1 for the absence of Rayleigh friction. In this condition, EVP-2 and EVP-1 are recognized as unstable modes. When Rayleigh friction is increased, EVP-2 and EVP-1 become singular in turn as depicted by the peaks in the response function. The EVP-1 is the singular eigenmode and becomes SVD-1 when the Rayleigh friction ν_S is 1.52×10^{-3} (e-folding damping time of 52 days).

Figure 2 illustrates the barotropic height of the EVP-1 which is easily recognized as the AO pattern with a negative pole over the Arctic and positive poles over Pacific and Atlantic. When the eigenvalue is zero, this mode becomes resonant and is excited dominantly by arbitrary steady forcing.

It is anticipated, however, that the Rayleigh friction of 52 days e-folding time seems too small compared with a typical value for the barotropic atmosphere. For this reason, we investigate the pair of right and left singular vectors for a typical value of the Rayleigh friction with 3.98×10^{-3} (e-folding time of 20 days) as noted by the right-most arrow in Fig. 1. The results of the barotropic height (rihgt singular vector) and the corresponding external forcing (left singular vector) for the SVD-1 under the strong Rayleigh friction are illustrated in Fig. 3 and Fig. 4, respectively. The barotropic height is still recognized as the AO pattern, although the Pacific positive pole is elongated in east-west direction, and the Atlantic positive pole moves eastward over Europe. When the Rayleigh friction is increased to 15 days e-folding time (not shown), the Pacific pole is separated in two poles over Japan and the west coast of the US, revealing a dominant zonal wavenubmer 3 patters, which may be difficult to recognize as the AO pattern.

The corresponding external forcing in Fig. 4 shows negative area in high latitudes and positive area in low latitudes with a pronounced positive peak over East Siberia. A zonal wavenumber 3 is superimposed on the dominant zonally symmetric pattern. When the Rayleigh friction is increased to 15 days e-folding time, the zonal wavenumber 3 dominates the zonal pattern (not shown).

In Fig. 5, the score time series of the ex-

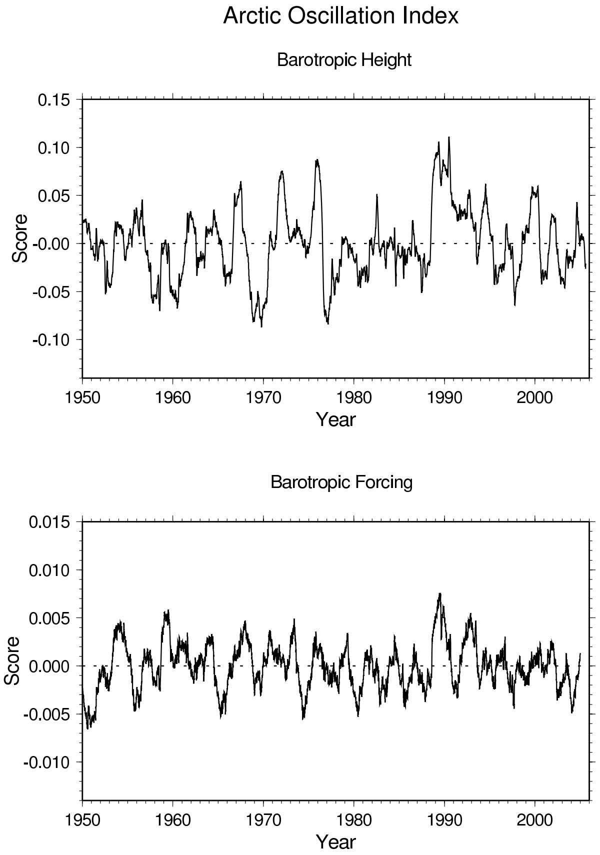

Figure 5: AO Index and the score time series of the external forcing by the NCEP/NCAR reanalysis projected on the SVD-1 in Fig. 4 for 1950 to 2004. Values are 365-day running mean.

ternal forcing, projected on the SVD-1 in Fig. 4 are investigated using the observed external forcing f_i in (2). Since (2) is exact within the truncation, we can obtain f_i as accurate as w_i by the residual balance of (2). The result of the annual running-mean is compared with the time series of the AO index.

For the time series of the SVD-1, the correlation coefficient with the AO index for 1950 to 2004 results in 0.35, which accounts for12% of the total variance of the AO index. The result is similar to the score time series projected onto the adjoint mode of the singular eigenmode, because the adjoint mode shows almost identical structure as in Fig. 4. Note that the external forcing f_i contains the damping terms. It is noted that the projection of the forcing onto the singular eigenmode in Fig. 2 shows high correlation as a result of damping term caused by the well established structure of the AO.

4. SUMMARY

In this study, the dynamics of the AO is examined from the view point of the neutral mode theory by Kimoto et al. (2001) and the singular eigenmode theory by Tanaka and Matsueda (2005). According to the result of the analysis, SVD-1 under a reasonably strong Rayleigh friction shows a structure similar to the observed AO. It is no doubt to understand that the AO is a dynamical normal mode of the global atmosphere. The characteristics of the neutral mode comes from that of the singular eigenmode. The SVD-1 forcing explains, however, only 12% of the total variance of the AO. Father study is required to investigate the mechanism to excite the neutral mode or singular eigenmode.

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REFERENCES

Kimoto, M., F.-F. Jin, M. Watanabe, and N. Yasutomi, 2001: Zonal-eddy coupling and a neutral mode theory for the Arctic Oscillation. . *Geophys. Res. Lett.*, **28**,737–740.

Tanaka, H.L., 2003: Analysis and modeling the Arctic Oscillation using a simple barotropic model with baroclinic eddy forcing, *J. Atmos. Sci.,* **60**, 1359-1379.

Tanaka, H.L. and M. Matsueda, 2005: Arctic oscillation analyzed as a singular eigenmode of the global atmosphere. *J. Meteor. Soc. Japan*, **83**, 611-619.

Tanaka, H.L. and K. Terasaki, 2005: Energy spectrum and energy flow of the Arctic Oscillation in the phase speed domain. *SOLA*, **1**, 65-68.

Watanabe, M. and F.-F. Jin, 2004: Dynamical prototype of the Arctic Oscillation as revealed by a neutral singular vector. *J. Clim.*, **17**, 2119–2138.