EOF and SVD Analyses of the Low-frequency Variability of the Barotropic Component of the Atmosphere

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1. Introduction

Low-frequency variability is a great concern in the extreme events (abnormal weather) of climate study as well as in the medium– to long–range weather forecasting. It has been well known that most of the low-frequency variability of the atmosphere such as blocking high, Arctic Oscillation (AO) and PNA-like teleconnections are characterized by their barotropic structure (Wallace and Blackmon 1981; Wallace and Gutzler 1981; Thompson and Wallace 1998; Tanaka 1998, 2003). The barotropic component of the atmosphere is dynamically unique in that it consists of an isolated discrete spectrum with respect to the vertical normal mode while all other baroclinic components construct a continuous spectrum (Staniforth et al. 1985). In this respect, we may regard the barotropic component as a physical mode of the atmosphere with characteristic behavior of the 2D fluid dynamics.

According to the analysis of the barotropic component by Tanaka and Matsueda (2004), more than 80 % of the recent extreme events are associated with the natural variability of the atmosphere under non-extreme external forcing in the Northern Hemisphere on the monthly mean time scale. Likewise, more than 80% of abnormal external forcing result in non–extreme weather. The result implies that the external forcing is the secondary importance for the occurence of the abnormal weather, which, in tern, emphasize the importance of the natural variability for the monthly mean time scale.

However, it is anticipated that an extreme event may be induced by a specific horizontal pattern of external forcing with rather small energy norm. There may be a characteristic combination of horizontal patterns between the atmosphere and its external forcing. According to Kimoto et al. (2001), a large atmospheric anomaly can be produced as a linear response to a small–normed external forcing when a least damping mode (neutral mode) is excited by the forcing. The least damping mode is derived as a singular vector with the least singular value of the dynamical system. Hence, it may be interesting to conduct SVD analysis for the pair of horizontal patterns of the atmosphere and its external forcing.

The purpose of this study is to conduct the EOF and SVD analyses for the horizontal patterns of the barotropic component of the atmosphere, its external forcing, and the nonlinear scale interactions. We focus on the anomaly of DJF–mean and DJF–daily data for the period 1952/53 through 2001/02, because the low-frequency variability dominates in DJF superimposed on a seasonal cycle.

2. Methods and Data

2.1 Methods

The analysis methods is based on the barotropic P–model described in Tanaka and Nohara (2001) where the external forcing of the barotropic model is evaluated from observations as a residual of the governing equation.

By expanding the state variable vectors $\mathbf{U} = (u,\ v,\ \phi')^T$ in 3–D normal mode functions, we obtain a system of 3–D spectral primitive equations in terms of the spectral expansion coefficients w_i, (see Tanaka and Matsueda 2004) :

$$\frac{dw_i}{d\tau} + i\sigma_i w_i = -i\sum_{jk} r_{ijk} w_j w_k + s_i, \qquad (1)$$

where w_i and s_i are 3–D normal mode expansion coefficients of the atmosphere and the external forcing, respectively, τ is a dimensionless time scaled by $(2\Omega)^{-1}$, σ_i is the eigenfrequency of the Laplace's tidal equation, and r_{ijk} is the interaction coefficients for nonlinear wave–wave interactions calculated by the triple products of the 3-D normal mode functions. The external forcing s_i is diagnostically calculated by (1) as the residual of the equation from w_i. Also, the barotropic–baroclinic interaction appears on the right–hand side as a part of s_i. The EOF and SVD analyses are performed for anomalies of variables w_i and s_i in the spectral domain. This equation is a simple spectral barotropic model, using only the barotropic components ($m = 0$) of the Rossby modes, by truncating all the baroclinic modes and high–frequency gravity modes.

As discussed by Tanaka and Matsueda (2004), total energy E of the atmosphere (sum of kinetic energy and available potential energy) is simply the sum of the energy elements E_i defined by:

$$E_i = \frac{1}{2} p_s h_m |w_i|^2, \tag{2}$$

where w_i is the state variable in (1), p_s is the mean surface pressure, and h_m is the equivalent depth of the vertical mode m. The value must be divided by 2 for zonal components.

The energy equation derived from (1) and (2) may be simply represented by the following equation:

$$\begin{aligned} \frac{dE_i}{dt} &= p_s h_m \Omega (w_i n_i^* + w_i^* n_i) \\ &+ p_s h_m \Omega (w_i s_i^* + w_i^* s_i), \end{aligned} \tag{3}$$

where n_i represents the nonlinear term in (1), and the superscript $(\ )^*$ denotes complex conjugate. This equation represents the energy variation for the barotropic component caused by the nonlinear term and the external forcing. The linear term in (1) does not contribute to the energy equation because it does not change the magnitude. Using this equation, we can quantitatively evaluate whether a given external forcing or the nonlinear scale interactions amplify the barotropic component or not.

For the analysis of anomaly, the variables w_i, n_i, and s_i in the energy equation are replaced by their anomalies w_i', n_i', and s_i', respectively. With this energy equation, we can evaluate the norm variation for a pair of two distinct variables of w_i' and s_i' and for a pair of two variables of w_i' and n_i' by the SVD analysis.

2.2 Data

The data used in this study are NCEP/NCAR reanalysis for 50 years from December 1952 to November 2002 (Kalnay et al. 1996). The data contain horizontal winds $\mathbf{V} = (u,\ v)$ and geopotential ϕ, defined at every 2.5° longitude by 2.5° latitude grid points over 17 mandatory vertical levels from 1000 to 10 hPa.

4. EOF analyses for DJF

The horizontal pattern of the first EOF (25.3% of total variance, not shown) appears to indicate the Arctic Oscillation (Thompson and Wallace, 1998). The second EOF (14.5%, not shown) looks like the Pacific/North American (PNA) teleconnection pattern (Wallace and Gutzler, 1981).

Also, The horizontal pattern of the EOF-1 of the external forcing(10.2%, not shown) has the structure of approximately wavenumber 1 with extremum at the central Eurasia. The horizontal pattern of the second EOF (8.1%, not shown) represents a features of topographic forcing.

5. SVD analyses for DJF

Next, SVD analyses are conducted for the barotropic component of the atmosphere and its external forcing, and for the barotropic component of the atmosphere and the nonlinear term in the Northern Hemisphere during DJF, in order to investigate statistical relationship between them.

Figure 1 illustrates the singular vectors and the time series of coefficients for the SVD-1 (35.8%). According to the result of the analysis, the atmospheric field in Fig. 1 (a) represents the AO pattern as seen for the EOF-1. The forcing field in Fig. 1 (b) looks like the topographic effect as a combination of horizontal patterns of the EOF-1 and EOF-2 of the forcing.

Although Figs. 1 (a) and (b) are most relevant patterns, it is found that the external forcing mainly works as a damping to the atmospheric anomaly in the seasonal time scale, because the norm variation by the external forcing for the SVD-1 is negative in the energy equation of (3). The same results are obtained for the SVD analyses applied to the monthly means of January, February, and March (not shown). These results suggest that the extreme event by the low-frequency variability is not excited by the low-frequency external forcing, but is caused by the nonlinear scale interactions in (3). Although the results are not shown, it is found that the external forcing up to ninth SVD works as damping. The result implies that the extreme anomaly by the low-frequency variability may be induced by the nonlinear term rather than the low-frequency external forcing.

In order to verify this hypothesis, we have similarly applied the SVD analysis for the barotropic component of the atmosphere and the nonlinear term. Consequently, we obtained the SVD pattern, where the energy conversion caused by the nonlinear term indicates positive value.

The energy tendency is nearly zero for the seasonal time scale, so the energy supply by the nonlinear interactions is balanced with dissipation by the external forcing.

6. SVD analysis for daily data

In this section, we have applied the SVD analysis for the daily data in order to investigate the importance of daily transient eddies for the low-frequency variability.

The first and second SVD patterns of the atmosphere and the external forcing for the daily data (not shown) look like the result of DJF in Fig. 1. This implies that the low-frequency variability dominates the transient eddy activities even for the daily data, showing the relation of energy sink patterns. For this reason, the SVD analysis is applied for 30-day high-pass filtered data, eliminating the low-frequency variability to confirm the importance of daily transient eddies. Figures 2 (a) and (b) illustrate the resulting singular vectors of the first SVD (12.7%), which have the structures of wavenumber 6 with well-defined extremum at the North Pacific and North America. The energy conversion caused by the external forcing indicates positive value as inferred from the patterns. The second SVDs (10.5%, not shown) are almost out of phase by a quarter of the wavelength and the energy conversion turns out to be positive as in SVD-1.

Next, we have applied the SVD analysis for the barotropic component of the atmosphere and the nonlinear term for 30-day high-pass filtered data, in order to investigate the role of nonlinear scale interactions with daily transient eddies. Figures 2 (c) and (d) illustrate the resulting singular vectors of the first SVD (15.9%), which have the well-defined extremum at North America and the Atlantic. The SVD-1 of the external forcing has well-defined signals at the Pacific, whereas the SVD-1 of the nonlinear term has a well-defined signal at the Atlantic. The nonlinear scale interactions damp the atmospheric anomaly for the daily data because the energy conversion caused by the nonlinear term indicates a negative value. The second SVDs (13.3%, not shown) represents a pattern of a quarter wavelength out of phase and the energy conversion appears to be negative as in SVD-1.

7. Conclusions and discussion

In this study, we investigated the dominant horizontal patterns of the barotropic component of the atmosphere, its external forcing, and the nonlinear scale interactions, using the NCEP/NCAR reanalysis for 50 years from 1952 to 2002.

First, we applied the EOF analysis for the DJF mean. The horizontal patterns of the EOF-1 and EOF-2 of the atmosphere represents an AO pattern and a PNA teleconnection pattern, respectively. the EOF-2 of the external forcing shows the characteristics of topographic forcing. On the other hand, the SVD-1 represents a structure of AO with the external forcing characterized by the topographic forcing as the combination of the EOF-1 and EOF-2 of the forcing. According to the analysis of energy budget, however, it is found that the external forcing of the SVD-1 damps the AO-like anomaly of the barotropic component. Also, the external forcing up to ninth SVD damps the atmosphere.

Next, we have similarly applied the SVD analysis for the barotropic component of the atmosphere and the nonlinear term. The energy conversion caused by the nonlinear term indicates positive value. From this results, it is shown that the nonlinear scale interactions mainly excite the atmospheric anomaly in the seasonal time scale.

Furthermore, we apply the SVD analysis for a 30-day high-pass filtered daily data during DJF from 1952/53 to 2001/02. We find that the most dominant variability is the baroclinic eddies induced by baroclinic instability with zonal wavenumber 6 showing positive energy conversion by the forcing. Also, it is found that nonlinear scale interactions damp the atmospheric anomaly.

According to the SVD analysis of the barotropic component of the atmosphere in this study, it is concluded that (1) the daily external forcing amplifies the transient eddies at the synoptic scale and the daily nonlinear scale interactions damps them; (2) in the monthly to seasonal mean time scale the nonlinear scale interactions excite the atmospheric anomaly and the external forcing works as damping of the atmospheric anomaly ; (3) the anomaly of the extreme event in low-frequency variabilities is likely to be induced by the nonlinear scale interactions from transient eddies at the synoptic scale rather than the low-frequency external forcing.

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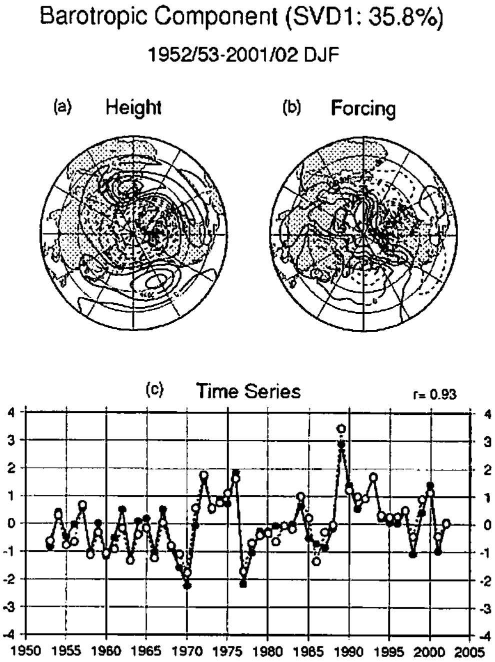

Figure 1. Horizontal patterns of the first SVD for the barotropic component of (a) the atmosphere and (b) the external forcing during DJF, and (c) the corresponding time series of coefficients (solid line for height, dashed line for forcing).

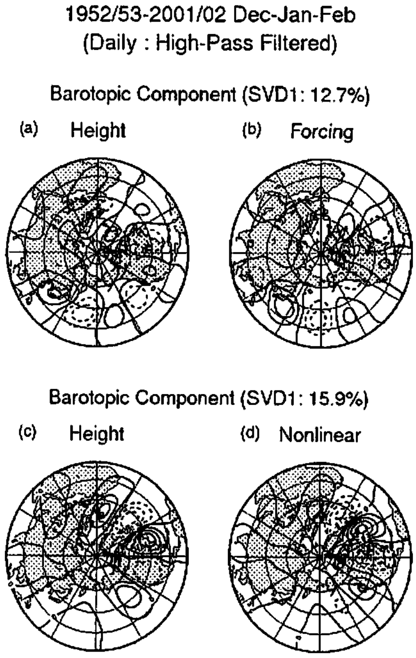

Figure 2. Horizontal patterns of the first SVD for the daily barotropic component of (a) the atmosphere and (b) the external forcing, and that of the first SVD for (c) the atmosphere and (d) the nonlinear scale interactions for the 30-day high-pass filtered data during DJF.