Energy Spectrum and Energy Flow for the Arctic Oscillation in the Phase Speed Domain

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1. INTRODUCTION

The Arctic Oscillation (AO) postulated by Thompson and Wallace (1998) has attracted more attention in recent years. The AO is a north-south seesaw of the atmospheric mass between the Arctic region and a surrounding zonal ring in the mid-latitudes, and has an equivalent barotropic structure from the surface to the lower stratosphere.

The spectral characteristics of the AO are described by Tanaka (2003) by means of the 3D normal mode decomposition, including the vertical spectrum. The analysis scheme is referred to as normal mode energetics. In the analysis, the scale of the 3D normal mode is represented by the eigen frequency of Laplace's tidal equation σ instead of the wavenumber k. The modal frequency is related to the scale by the wave dispersion relation.

The spectral peak for eddies over the phase speed domain (i.e., c domain) is explained by the Rhines (1975) scale which separates the distinct slopes of turbulence and wave regimes (see Fig. 1). The spectral slope in the phase speed domain is theoretically deduced by Tanaka et al. (2004) to establish the energy spectrum of c^2 $E = mc^2$, based on Garcia's (1991) criterion of Rossby wave breaking $\partial q/\partial y < 0$. When the Rossby waves saturates in the turbulence regime, the excessive energy accumulated at the synoptic eddies cascades up toward the spherical Rhines speed c_R. The accumulated energy at c_R would stay for long time because there is no way to break down the amplified Rossby wave by the triad wave-wave interactions of turbulence. Tanaka and Terasaki (2004) postulated that the atmospheric blocking is formed when excessive energy is accumulated at the spherical Rhines speed c_R exceeding the Rossby wave saturation theory.

Similar analogy of the energy flow in the phase speed domain will lead to a hypothesis such that the accumulated energy at the spherical Rhines speed c_R is transferred to the zonal flow by the zonal-wave interaction, creating the Arctic Oscillation.

The purpose of this study is first to examine the up-scale energy cascade from the synoptic eddies to the zonal field in the phase speed domain. Second, we confirm our speculation such that the AO is characterized as the accumulation of barotropic energy at a specific meridional mode of the zonal field. Finally, we confirm the intensification of the zonal-wave interaction during the AO positive phase by the composite analysis.

2. EQUATION AND DATA

By expanding the state variable in 3D normal mode functions, we obtain a system of 3D spectral primitive equations in terms of the spectral expansion coefficients w_i, (see Tanaka and Terasaki 2004):

$$\frac{dw_i}{d\tau} = -i\sigma_i w_i - i\sum_{jk} r_{ijk} w_j w_k + f_i, \qquad (1)$$

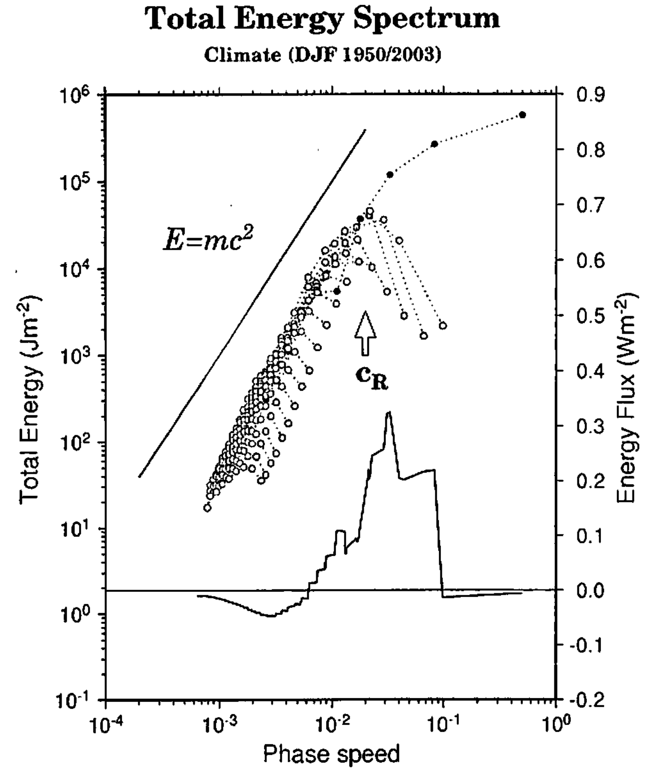

Figure 1: Barotropic energy spectrum E_i and the energy flux F_i as a function of the dimensionless phase speed of Rossby waves $|c_i|$. The spherical Rhines speed c_R is marked by an arrow.

where τ is a dimensionless time, σ_i is the eigenfrequency of the Laplace's tidal equation, f_i is the expansion coefficient of the external forcing of viscosity and diabatic heating rate, and r_{ijk} is the interaction coefficients for nonlinear wave-wave interactions calculated by the triple products of the 3D normal mode functions.

Total energy E of the atmosphere (sum of kinetic energy and available potential energy) is simply the sum of the energy elements E_i defined by:

$$E_i = \frac{1}{2} p_s h_m |w_i|^2, \tag{2}$$

where p_s is the mean surface pressure and h_m is the equivalent height. The energy spectrum E_i is plotted as a function of the dimensionless phase speed of the Rossby mode $c_i = \sigma_i/n$ in a resting atmosphere, where n is the zonal wavenumber. The phase speed c_i represents the horizontal scale of a mode by the wave dispersion relation. The westward phase speed is small (large) when the horizontal scale of the Rossby mode is small (large).

By differentiating (2) with respect to time and substituting (1), we obtain the energy balance equation:

$$\frac{dE_i}{dt} = N_i + S_i, \tag{3}$$

where N_i and S_i designated the nonlinear interactions and the energy sources, respectively. We then define energy flux in the phase speed domain F_i by the summation of the nonlinear interactions N_i with respect to c_i in descending order of magnitude:

$$F_i = \sum_{k=1}^{i} N_k. \tag{4}$$

The energy flux is further decomposed in contributions from zonal-wave interactions F_{Zi} and wave-wave interactions F_{Wi}.

The data used in this study are four-times daily NCEP/NCAR reanalysis for 51 years from 1950 to 2000 (see Kalnay et al. 1996). The data contain horizontal winds (u, v) and geopotential ϕ, defined at every 2.5° longitude by 2.5° latitude grid point over 17 mandatory vertical levels from 1000 to 10 hPa.

3. ENERGY SPECTRUM

Figure 1 illustrates the barotropic energy spectrum E_i and the energy flux F_i as a function of c_i. Energy levels are connected by dotted lines for the same zonal wavenumber n with different meridional mode numbers l.

For zonal wavenumber zero, the scale index is not defined because Laplace's tidal equation degenerates for geostrophic modes. The difficulty was overcome by Shigehisa (1983) where mathematical limits of $c_i = \sigma_i/n$ are shown to converge to finite values. The phase speed of the geostrophic mode can be approximated by that of the Haurwitz wave on a sphere:

$$c_i = \sigma_i/n \approx \frac{-1}{l(l+1)}, \tag{5}$$

where l is the meridional mode number of n=0. Using this definition of the phase speed, we can analyze the energy spectrum for all zonal waves, including n=0.

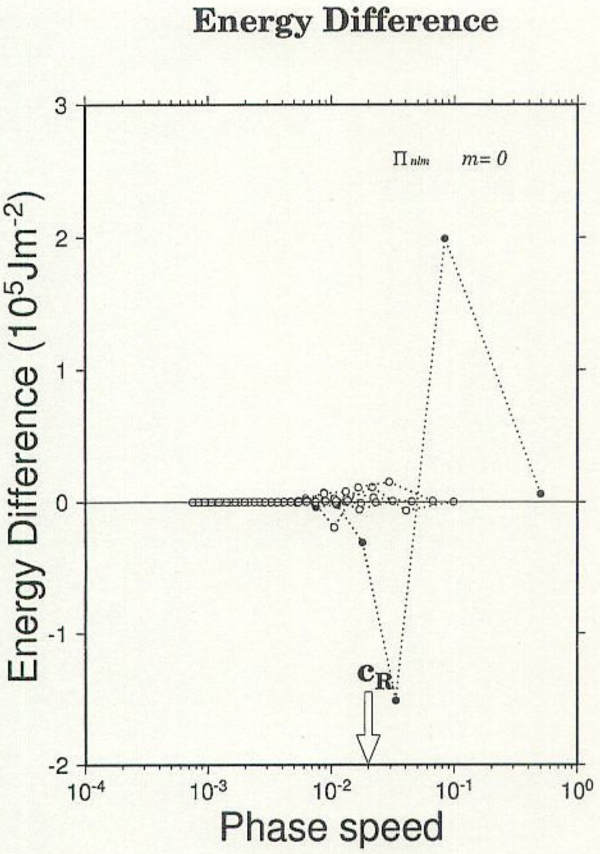

Figure 2: Distribution of barotropic energy difference for the AO positive subtracted by AO negative in the phase speed domain.

The energy spectrum for n=0 over c_i appears to coincide with that of $n \neq 0$ for the small meridional scale. The energy injected at the synoptic scale (small c_i) cascades up to the larger scale (larger c_i) obeying a specific power law. The spectral peak at c_i=0.02 for eddies is clearly explained by the spherical Rhines speed c_R which separates the turbulence regime and wave regime. The spherical Rhines speed is also the speed where the westward phase speed of the Rossby wave becomes stationary, and appreciable amount of energy supply occurs by the topographic forcing. The line in the figure denotes the spectral slope of $E = mc^2$ derived by Tanaka et al. (2004) from the Rossby wave saturation criterion $\partial q/\partial y < 0$, where $m = p_s/g$ is the atmospheric mass in unit area. The theoretical slope agrees well with the observation even for n=0.

Figure 1 illustrates also the energy flux F_i in the phase speed domain. According to the result, the energy flux diverges at the synoptic scale and cascades up toward larger scale beyond c_R showing the peak value of 0.30 W m^{-2}. The up-scale energy flux converges at c_i=0.1, which corresponds to l=3 of n=0.

4. Arctic Oscillation

Figure 2 illustrates the distribution of barotropic energy difference for the AO positive subtracted by AO negative in the phase speed domain. The AO positive and AO negative are the composite of the AO time series for the standard deviation ±1.5 and above, respectively. It is found that the AO is characterized by the accumulation of energy at l=3 and reduction at l=5 of the zonal field. The positive and negative peaks reach 2.0 and -1.5 × 10^5 J m^{-2}, respectively.

Figure 3 shows the energy flux associated with the zonal-wave interactions F_Zi in the phase speed domain for the composite of the AO positive (solid line) and AO negative (dashed line) compared with the climate (dotted line). As discussed by Tanaka and Terasaki (2004), the up-scale energy flux from the synoptic-scale source range F_Wi converges at c_R during blocking events. It is noteworthy that the accumulated energy at c_R is further transferred to l=3 by the energy flux F_Zi. The excessive energy at l=3 is evidently resulted from the energy flux convergence at l=3. The reduction of energy at l=5 is also explained by the reduced flux convergence at l=5. It is confirmed that the up-scale energy flux F_Zi is clearly instrumental for the AO.

5. CONCLUSION

In this study, energy spectrum of the large-scale atmospheric motions is examined in the framework of the 3D normal mode decomposition. Attention is concentrated to the barotropic component of the atmosphere where low-frequency variability dominates.

According to the result of the observational

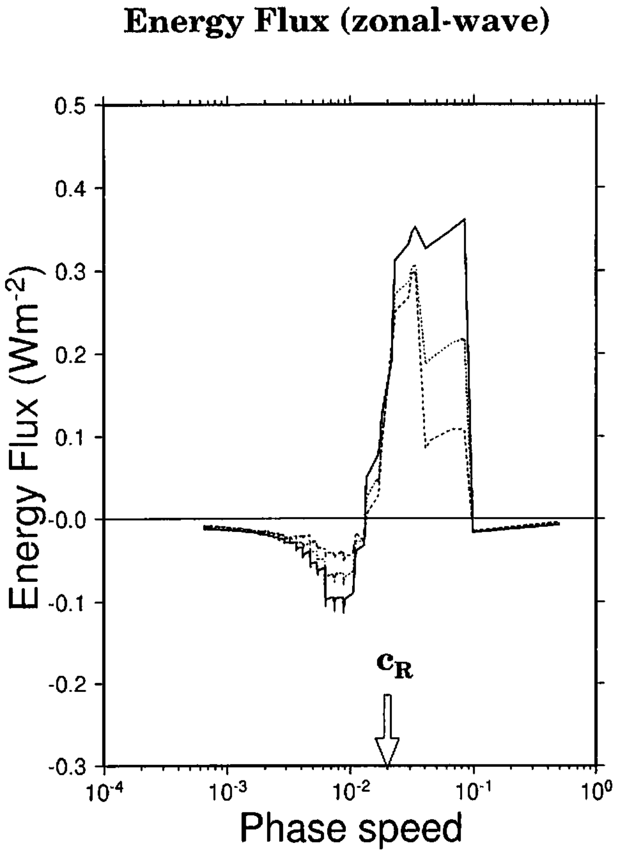

Figure 3: Energy flux in the phase speed domain associated with the zonal-wave interactions evaluated for the AO positive (solid line), AO negative (dashed line) and climate (dotted line).

analysis, the AO in the phase speed domain is represented by the energy increase at l=3 mode with simultaneous decrease at l=5 of the zonal field. The energy accumulation at l=3 is explained by the enhanced energy flux $F_Z i$ associated with the zonal-wave interaction.

It is found in this study that the energy flux $F_Z i$ comes from the spherical Rhines speed c_R where the planetary-scale Rossby waves are stationary. Kimoto et al. (2001) and Watanabe and Jin (2004) suggested that the AO is induced by the interactions with the forced steady planetary waves. In contrast, Tanaka (2003) suggested that the AO is induced by the interactions with the active synoptic eddies. It is shown in this study that the up-scale energy flux by transient eddies is once accumulated at c_R by $F_W i$. The accumulated energy is then transferred to zonal field by $F_Z i$ to cause the AO. The result shows that the low-frequency variability associated with the AO is maintained by energy flux from c_R, which is compensated by the up-scale cascade from synoptic eddies rather than the forced steady planetary waves.

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