Analysis and Modeling of the Arctic Oscillation Using a Simple Barotropic Model with Baroclinic Eddy Forcing

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ABSTRACT

In this study, a numerical simulation of the Arctic Oscillation (AO) is conducted using a simple barotropic model that considers the barotropic–baroclinic interactions as the external forcing. The model is referred to as a barotropic S model since the external forcing is obtained statistically from the long-term historical data, solving an inverse problem. The barotropic S model has been integrated for 51 years under a perpetual January condition and the dominant empirical orthogonal function (EOF) modes in the model have been analyzed. The results are compared with the EOF analysis of the barotropic component of the real atmosphere based on the daily NCEP-NCAR reanalysis for 50 yr from 1950 to 1999.

According to the result, the first EOF of the model atmosphere appears to be the AO similar to the observation. The annular structure of the AO and the two centers of action at Pacific and Atlantic are simulated nicely by the barotropic S model. Therefore, the atmospheric low-frequency variabilities have been captured satisfactorily even by the simple barotropic model.

The EOF analysis is further conducted to the external forcing of the barotropic S model. The structure of the dominant forcing shows the characteristics of synoptic-scale disturbances of zonal wavenumber 6 along the Pacific storm track. The forcing is induced by the barotropic–baroclinic interactions associated with baroclinic instability.

The result suggests that the AO can be understood as the natural variability of the barotropic component of the atmosphere induced by the inherent barotropic dynamics, which is forced by the barotropic–baroclinic interactions. The fluctuating upscale energy cascade from planetary waves and synoptic disturbances to the zonal motion plays the key role for the excitation of the AO.

1. Introduction

The Arctic Oscillation (AO) postulated by Thompson and Wallace (1998, 2000) has attracted more attention in recent years. The AO is a north–south seesaw of the atmospheric mass between the Arctic region poleward of 60°N and a surrounding zonal ring in the midlatitudes. It is defined as a primary mode of an empirical orthogonal function (EOF) for the sea level pressure field in the Northern Hemisphere. The spatial pattern of the AO is characterized by its zonally symmetric or “annular” structure centered in the Arctic. The AO bears a superficial resemblance to the leading mode of low-frequency variability in the Southern Hemisphere, which is now referred to as a Southern Hemisphere annular mode (SAM). In this regard, a terminology of Northern Hemisphere annular mode (NAM) is preferred for the AO by Wallace (2000). In this study, we will refer to the AO as the EOF-1 of atmospheric variability with one sign over the Arctic and two poles of opposite sign over the Pacific and Atlantic sectors, while the terminology of the annular mode is reserved essentially for the zonally symmetric variation in the atmosphere.

The AO is successfully simulated by a number of realistic general circulation models with fixed forcing (Feldstein and Lee 1998; Yamazaki and Shinya 1999; Fyfe et al. 1999; Shindell et al. 1999; Limpasuvan and Hartmann 1999, 2000; Boer et al. 2001; Robertson 2001). The basic features of the zonally symmetric structures are also simulated using simple quasigeostrophic and primitive equation models with simple physics and no topography or seasonal cycle (Robinson 1991, 1994, 1996; James and James 1992; Yu and Hartmann 1993; Lee and Feldstein 1996; Feldstein and Lee 1998; Akahori and Yoden 1997). Because the external forcing of the model is independent of time, the variability found in these simple models is the unforced variability associated with dynamical processes internal to the atmosphere (Feldstein 2002). The dynamical examination of the annular modes shows that the baroclinic eddies in midlatitudes played an important role for the internal variability of the zonal mean flow through the positive feedback in momentum budget between the eddies and the mean flow (Karoly 1990; Shiotani

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been applied to the actual forecasting of the barotropic component of the atmosphere (Tanaka and Nohara 2001). We demonstrated that the model has a meaningful deterministic predictability of 8 days, which is comparable to the operational weather forecasting models.

In this study the model was integrated for 51 yr under a perpetual January condition to simulate the realistic pattern of the AO. The result of the EOF analysis of the model atmosphere was compared with the observed EOF patterns using the National Centers for Environmental Prediction (NCEP) and National Center for Atmospheric Research (NCAR) reanalysis for 50 yr from 1950 to 1999 (see Kalnay et al. 1996).

In section 2 the governing equations and the data used in this study are described. In section 3 the result of the EOF analysis is presented for the observed atmosphere in order to compare with that of the model atmosphere. The results of the 51-yr time integration of the model and the dominant EOF modes are presented in section 4. Finally, in section 5, the dynamical interpretation of the AO is discussed in reference to the energetics analysis of the AO.

2. Method and data

a. Governing equations

The model description is provided by Tanaka (1998) and Tanaka and Nohara (2001), so a brief description is presented here. A system of primitive equations with a spherical coordinate of longitude \( \lambda \), latitude \( \theta \), pressure \( p \), and time \( t \) may be reduced to three prognostic equations of horizontal motions and thermodynamics for three dependent variables of \( U = (u, v, \phi') \). Here, \( u \) and \( v \) are the zonal and meridional components of the horizontal velocity, respectively, and \( \phi' \) is a departure of the local isobaric geopotential from the reference state geopotential \( \phi_0 \). The superscript “’” denotes a transpose. Using a matrix notation, these primitive equations may be written as

\[
M \frac{dU}{dt} + LU = N + F, \tag{1}
\]

where the left-hand side of (1) represents linear terms with matrix operators \( M \) and \( L \) and the dependent variable vector \( U \). Refer to Tanaka (1998) for the definition of matrices \( M \) and \( L \). The right-hand side represents a nonlinear term vector \( N \) and a diabatic term vector \( F \), which includes the zonal and meridional components of frictional forces and a diabatic heating rate.

In order to obtain a system of 3D spectral primitive equations, we expand the vectors \( U \) and \( F \) in 3D normal mode functions in a resting atmosphere, \( \Pi_{nlm}(\lambda, \theta, \rho) \):

\[
U(\lambda, \theta, p, t) = \sum_{nlm} w_{nlm}(t) X_n \Pi_{nlm}(\lambda, \theta, \rho), \tag{2}
\]

\[
F(\lambda, \theta, p, t) = \sum_{nlm} f_{nlm}(t) Y_m \Pi_{nlm}(\lambda, \theta, \rho), \tag{3}
\]
where the dimensionless expansion coefficients \( w_{\lambda\theta p}(t) \) and \( f_{\lambda\theta p}(t) \) are the functions of time alone. These may be computed by the inverse Fourier transforms of \( U \) and \( F \) from the observed data.

\[
w_{\lambda\theta p}(t) = (U(\lambda, \theta, p, t), X_\lambda^m \Pi_{\lambda\theta p}(\lambda, \theta, p)).
\]

\[
f_{\lambda\theta p}(t) = (F(\lambda, \theta, p, t), Y_\lambda^m \Pi_{\lambda\theta p}(\lambda, \theta, p)).
\]

The subscripts represent zonal wavenumbers \( n \), meridional indices \( l \), and vertical indices \( m \). The scaling matrices \( X_\lambda^m \) and \( Y_\lambda^m \) are defined at each vertical index. The expansion basis of the 3D normal mode functions \( \Pi_{\lambda\theta}(\lambda, \theta, p) \) is obtained as an eigensolution of a homogeneous partial differential equation, putting zero on the right-hand side of (1). The 3D normal mode functions are given by a tensor product of vertical structure functions \( G_n(p) \) and Hough harmonics \( H_{\lambda\theta}(\lambda, \theta) \) associated with the linear operators \( M \) and \( L \), respectively (Kasahara 1976, 1978). They form a complete set and satisfy an orthonormality condition under a proper inner product (\( \cdot, \cdot \)).

By expanding those variables in 3D normal mode functions, we obtain a system of 3D spectral primitive equations in terms of the spectral expansion coefficients:

\[
\frac{dw_i}{d\tau} + i\sigma_i w_i = -i \sum_{jk} r_{ij} w_j w_k + f_i,
\]

\[
i = 1, 2, 3, \ldots ,
\]

where \( \tau \) is a dimensionless time scaled by \((2\Omega)^{-1}\), \( \Omega \) is the angular speed of the earth’s rotation, \( \sigma_i \) is the eigenfrequency of the Laplace’s tidal equation, and \( r_{ij} \) is the interaction coefficients for nonlinear wave–wave interactions calculated by the triple products of the 3D normal mode functions. The triple subscripts are shortened for simplicity as \( w_{\lambda\theta p} = w_i \). There should be no confusion in the use of \( i \) for a subscript even though it is used for the imaginary unit in (6).

b. Construction of a barotropic S model

In the 3D spectral representation, the vertical expansion basis functions may be divided into barotropic \((m = 0)\) and baroclinic \((m \neq 0)\) components. We may construct a simple spectral barotropic model, using only the barotropic components \((m = 0)\) of the Rossby modes, by truncating all the baroclinic modes and high-frequency gravity modes (see Kasahara 1977). Such a model is equivalent to a model predicting the vertical average of meteorological variables. The barotropic components capture the essential features of the low-frequency variability of planetary-scale motions. The spectral equation for such a barotropic model may be written as

\[
\frac{dw_i}{d\tau} + i\sigma_i w_i = -i \sum_{jk} r_{ij} w_j w_k + f_i + g_i,
\]

\[
i = 1, 2, 3, \ldots , \quad (m = 0),
\]

where the indices of the subscripts run only for the barotropic modes. The zonal and meridional wave truncation of the present model is equivalent to rhomboidal 20 with an equatorial wall. The degree of freedom of the system is reduced enormously by these truncations. The spectral equation for such a barotropic model \((7)\) has the same form as for the baroclinic model equation \((6)\), except for the fact that the barotropic–baroclinic interaction \( g_i \) appears on the right-hand side. Henceforth, we combine \( f_i + g_i \) and designate it as the external forcing \( s_i \) of the barotropic model.

The parameterization of the external forcing \( s_i = f_i + g_i \) in \((7)\) is presented by Tanaka (1991, 1998) considering the dynamical processes of the baroclinic instability, the topographic forcing, the biharmonic diffusion, the zonal surface stress, and the Ekman pumping. Since those parameterizations are fundamentally linear approximations to the true forcing and will never be a perfect representation of the real atmosphere, we have attempted in this study to obtain the best linear fit of the forcing \( s_i \) based on the 50 yr of the NCEP–NCAR reanalysis from 1950 to 1999 as in Tanaka and Nohara (2001):

\[
s_i = \bar{s}_i + s'_i = \bar{s}_i + A_{ij} w_j + B_{ij} w^*_j + e_i,
\]

where \( \bar{s}_i \) and \( s'_i \) are the climate and anomaly of \( s_i \), respectively. The asterisk for \( w_i \) represents a complex conjugate of \( w_i \). In general, \( w_i \) and \( w^*_i \) are linearly independent, and both are required in the complex-valued regression since only the positive (and zero) zonal wavenumbers are considered for the state variables of the model. We can evaluate the perfect external forcing of \( s_i \) from the observation as the residual of Eq. (7). The climate \( \bar{s}_i \) is obtained by the time mean of \( s_i \) for the 50-yr period. The first four harmonics of the seasonal change are synthesized to get a smooth seasonal march of \( \bar{s}_i \), and the anomaly \( s'_i \) is evaluated as the deviation from it.

The unknown linear matrices \( A_{ij} \) and \( B_{ij} \) can be obtained by the standard method of the least square fitting for \( s'_i \) to minimize the regression residual. In the stepwise least square fitting method, the matrix \( A_{ij} \) is evaluated first by

\[
A_{ij} = \bar{s}'_j / w^*_j,
\]

where \( s'_i = s_i - \bar{s}_i \) \((\bar{s}) \) is the time mean, and \( w^*_j \) is referred to as the pseudoinverse of \( w_j \) defined by

\[
w^*_j = w^H_j (w_j w^H_j)^{-1},
\]

where the superscript \( H \) denotes a complex conjugate transpose. It is a linear inverse problem (see Winkler et al. 2001) to obtain the system matrix from the known data of \( w_i \) and \( s'_i \). Similarly, the matrix \( B_{ij} \) may be obtained by minimizing the remainder \( e_i \) by regressing the first residual \( \bar{s}_i \), which is obtained by the first regression associated with \( A_{ij} \):

\[
B_{ij} = \delta_j w^*_j,
\]

using the pseudoinverse of \( w^*_j \).

The model with such a forcing is named "barotropic
S model" since the forcing is obtained statistically from observation, solving the inverse problem. The application of the barotropic S model to real forecasting is demonstrated by Tanaka and Nohara (2001), showing the mean predictability of 8 days according to the anomaly correlation. Although the barotropic S model demonstrates considerable predictability, the synoptic eddies appear to be weak by this formulation. The system matrices $A_y$ and $B_y$ mostly represent topographic forcing (TF). For this reason, we have added the energy source in terms of the parameterized baroclinic instability (BC) described by Tanaka (1998). The increased energy source at the synoptic scale requires an additional energy sink, so the biharmonic diffusion (DF), the zonal surface stress (DZ), and the Ekman pumping (DE) are also included in reference to the resulting energy spectrum of the model climate. The final form of the external forcing $s_i$ is expressed as

$$s_i = \tilde{s}_i + A_{ij}w_j + B_{ij}w_j^* + (BC)_{ij}w_j + (DF)_{ij}w_j + (DZ)_{ij}w_j + (DE)_{ij}w_j.$$  

Note that the topographic forcing is contained in $\tilde{s}_i$, as well as the system matrices. Although the barotropic–baroclinic interaction is implicitly considered in the model, the forcing has been modeled entirely by the barotropic component of the state variables $w$. Hence, any variability generated within the model may be considered as a natural variability governed by the barotropic dynamics internal to the model atmosphere.

c. Data and procedures

The data used in this study are four-times-daily NCEP–NCAR reanalysis for 51 yr from 1950 to 2000 (see Kalnay et al. 1996). The data contain horizontal winds $V = (u, v)$ and geopotential $\phi$, defined at every 2.5° latitude by 2.5° longitude grid point over 17 mandatory vertical levels from 1000 to 10 hPa.

The expansion coefficients $w_i$ are obtained by the inverse Fourier transform of (4) from the dataset of $U = (u, v, \phi')$. Although the data are recorded 4 times
Total Energy Spectrum
NCEP/NCAR DJF 1950-1999

Fig. 2. Barotropic energy spectrum of $E_m$, as a function of the dimensionless scale index $|c_{nm}|$, evaluated for the 50 yr of the NCEP–NCAR reanalysis during winter. The black and white dots denote $n = 0$ and $n \neq 0$, respectively. (Unit: J m$^{-2}$.)

daily, we refer to it as daily data in subsequent sections since the diurnal variation of $w$, is removed by a time filter. The external forcing $s = f_i + g_i$ is then diagnostically calculated by (7) as the residual of the equation, which provides the climate $\bar{s}$, and the anomaly $s'$ in (8). The EOF analysis is applied to the time series of $w$, and $s$, for the 50-yr data from 1950 to 1999 to find the dominant mode in the real atmosphere. With the known $w$, and $s$, the linear matrices $A_m$ and $B_m$ in (8) are computed by (9) and (11) based on the data for December–January–February (DJF) during the 50 yr. The year 2000 is reserved for a statistically independent forecasting as demonstrated by Tanaka and Nohara (2001). Finally, the barotropic $S$ model of (7) is integrated for 51 yr under a perpetual January condition, starting from the initial data of $w$, at 0000 UTC on 1 January 1950 with the value of $\bar{s}$ fixed on 1 January.

3. Dominant modes of the observed atmosphere

a. Barotropic component of the atmosphere

In general, the low-frequency variabilities such as AO, North Atlantic Oscillation (NAO), and Pacific–North America (PNA) are characterized by the equivalent barotropic structure. In this study, the 3D structures of atmospheric variables are projected onto the barotropic component of the vertical normal mode functions:

$$ (u, v, \phi')_m = \frac{1}{P_s} \int_0^{P_s} (u, v, \phi')^T G_0 dp. \quad (13) $$
Global Total Energy
NCEP/NCAR DJF 1950-1999

Fig. 3. Barotropic energy spectrum as a function of the zonal wavenumber \( n \) by
summing all meridional indices in Fig. 2. (Unit: J m\(^{-2}\).)

Here, \( G_0 \) is the vertical normal mode of \( m = 0 \), and \( p_s \)
is the surface pressure of the reference state. It is a part of
the inverse Fourier transform of (4) associated with
the vertical transform. The vertical transform (13) is
followed by the Fourier–Hough transforms to obtain the
expansion coefficient \( W \) in (4). Since the structure of
\( G_0 \) is approximately constant with no node in the ver-
tical, the vertical transform for \( m = 0 \) may be regarded
as the vertical mean of the state variables. In this study,
the left-hand side of (13) is referred to as a barotropic
component of the atmosphere that depends only on lon-
gitude, latitude, and time.

Figure 1 illustrates the barotropic component of the
gopotential height averaged for the winter (DJF) from
1950 to 1999. In order to obtain this figure, the expan-
sion coefficients \( W \) are averaged for 50 yr in the spectral
domain, then the mean \( W \) is converted to the geopo-
tential field using Eq. (2). Since geopotential height is
defined by the deviation from the global mean as dis-
cussed in (1), the value is negative at the polar region
and is positive in the Tropics. We will refer to it as a
barotropic height.

In the spectral domain, total energy \( E \) (sum of kinetic
energy and available potential energy) is simply the sum
of the energy elements \( E_i \) defined by

\[
E_i = \frac{1}{2} \rho_n \| h_n \| W_i \|^2, \tag{14}
\]

where \( h_n \) is the equivalent depth of the vertical mode
\( m \). The 3D scale of the normal mode has been defined
in Tanaka (1985) by the eigenfrequency of Laplace's
tidal equation \( \sigma_i \) in (6). Especially for the Rossby mode,
the suitable choice of the scale index is the westward
phase speed $c_i = \sigma_i/n$. It is known from the Rossby wave dispersion that the larger the Rossby mode is, the larger the westward phase speed is. (There is an exception in planetary waves for the Hough modes.) Therefore, the westward phase speed $|c_i|$ represents the 3D scale of the Rossby mode. The energy spectrum of $E_i$ as a function of the scale index $|c_i|$ was analyzed by Tanaka and Kasahara (1992), showing that the spectrum over the scale index $|c_i|$ obeys a unique power law even for the zonal wavenumber zero. For zonal wavenumber zero, the scale index is not defined because Laplace’s tidal equation degenerates for Rossby modes. All eigenvalues of $\sigma_i$ are zero for $n = 0$. The difficulty was overcome by the use of Shigehisa modes (Shigehisa 1983) where mathematical limits of $c_i = \sigma_i/n$ converge to finite values. The energy spectrum for $n = 0$ over $|c_i|$ appears to coincide with that of $n \neq 0$ for the small meridional scale. It was demonstrated in Tanaka (1991) that the phase speed of the geostrophic mode proposed by Kasahara (1978), which is used in this study, can be approximated by that of the Haurwitz wave on a sphere:

$$c_i = \frac{\sigma_i}{n} = \frac{-1}{l(l+1)},$$

where $l$ is the meridional mode number of $n = 0$. Using this definition of the scale index $|c_i|$ for $n = 0$, we can analyze the energy spectrum for all zonal waves, including $n = 0$.

Figure 2 plots the barotropic ($m = 0$) energy spectrum of $E_i = E_{adm}$ as a function of the scale index $|c| = |c_{adm}|$ evaluated for the 50 yr of the NCEP–NCAR reanalysis during winter. The dotted lines connect spectra for the same zonal wavenumber $n$ over different meridional mode numbers $l$. The spectrum obeys approximately the third power of $|c|$, except for the largest-scale meridional modes, which indicate less energy levels. The peak of the energy spectrum for the zonal eddy $n \neq 0$ is basically determined by the Rhines’s scale, where the inverse energy cascade from short waves to planetary waves terminates (Rhines 1979). It is interesting to note that the spectrum for $n = 0$ coincides with that of $n \neq 0$ for a smaller $|c|$, and it continues to increase as the meridional index $l$ decreases. We can confirm that the scale index for $n = 0$ in (15) is a good approximation to the energy spectrum of the Shigehisa mode presented in Tanaka and Kasahara (1992).

Figure 3 plots the zonal energy spectrum for the barotropic component of the atmosphere as a function of the zonal wavenumber $n$ by summing all meridional indices in Fig. 2. The energy level is $1.3 \times 10^5 \text{ J m}^{-2}$ for $n = 1$. It drops to $2.4 \times 10^4 \text{ J m}^{-2}$ for $n = 20$. The zonal energy is $10.6 \times 10^5 \text{ J m}^{-2}$ (not shown). The energy spectrum is used as a reference when the barotropic S model is constructed.

Figure 4 plots the daily time series of total energy for the barotropic component of the atmosphere from 1990 to 2000. The solid and dashed lines represent zonal and eddy energy, respectively. The time series is simply the sum of $E_i$ for zonal wavenumber $n = 0$ and for $n \neq 0$ evaluated in the spectral domain. According to the result, barotropic zonal energy exceeds $10 \times 10^5 \text{ J m}^{-2}$ in winter, and eddy energy is about one-half of it. Barotropic energy is reduced to $2 \times 10^4 \text{ J m}^{-2}$ in summer. There are substantial interseasonal and interannual variability in $E_i$, and thus in $w_i$. 

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**Fig. 4.** Daily time series of barotropic energy from 1990 to 2000. The solid and dashed lines represent zonal ($n = 0$) and eddy ($n \neq 0$) energy, respectively. (Unit: $10^5 \text{ J m}^{-2}$.)
Fig. 5. First two EOFs components of the barotropic component of the observed atmosphere, analyzed by the state variable, w, with the daily NCEP-NCAR reanalysis from 1950 to 1999.
b. **EOF analysis of the observed atmosphere**

The climate of $w_i$ with its seasonal march is computed from 50 yr of the time series data. Only the first four harmonics of the seasonal change are retained to get the smooth seasonal change of the climate of $w_i$. Once the climate is constructed, the anomaly of $w_i$ is computed for the analysis period. The EOF analysis is then conducted for the anomaly time series of $w_i$ in the spectral domain based on the daily data. The complex numbers of $w_i$ are split in real and imaginary parts to compute the covariance matrix for the EOF analysis.

Figure 5 illustrates the first two EOF components of the atmosphere represented by the barotropic height. The structure of EOF-1 indicates a negative region in high latitudes surrounded by a positive region in the midlatitudes. The positive region shows two peaks over the Pacific and Atlantic sectors. The zero crossing is located around 60°N. The structure compares quite well with the AO analyzed by Thoimp-
Barotropic Component of Geopotential Height

EOF-1 (21.0%)

Barotropic Component of Geopotential Height

EOF-2 (10.3%)

Fig. 7. As in Fig. 5, but for the first two EOFs analyzed with the winter mean (DJF) data from 1950 to 1999.
Barotropic Component

Time Spectrum of AO Index

Fig. 8. The time spectrum of the daily AO index of EOF-1 in Fig. 6.

son and Wallace (1998), so EOF-1 of this study is easily identified as the AO. The original definition of the AO is based on sea level pressure, which is a single variable at a single level. It should be noted that our result is based on the whole column of vertical data using all state variables \((u, v, \phi)\) during 50 yr of the daily data. Although our EOF analysis is performed for the barotropic component from the beginning, EOF-1 should be recognized as the most dominant low-frequency variability in the 3D atmosphere since most of the low-frequency variabilities are contained in the barotropic component. Since the daily data contain active synoptic-scale eddies, it may be worth noting that the AO remains the most dominant mode even for the analysis with the daily data.

EOF-1 explains 5.7\% of the total variance of the daily data. The score time series (365-day running mean) of the EOFs are plotted in Fig. 6. The time series of EOF-1 is referred to as the AO index, which represents considerable interannual variabilities. The positive value corresponds to a strong polar vortex while the negative value indicates a weak polar vortex. The AO index fluctuates between positive and negative values for 1950 to 1970, and it remains negative from 1976 to 1989. A notable shift from negative to positive occurs in 1989. The positive AO index then gradually decreases to a negative AO in the late 1990s, but it returns to positive in 2000. Clear decadal variability is contained in the AO index, but no significant trend is detected for the barotropic component of the atmosphere. According to the original definition of the AO index by Thompson and Wallace (1998), the decadal variability is superimposed on a recent trend related to global warming. Such a trend may be characterized in the baroclinic component of the atmosphere.

EOF-2 explains 4.3\% of the total variance of the daily data. A negative area over the Arctic is surrounded by positive values in the midlatitudes, with three peaks over the Atlantic Ocean, Siberia, and Canada. The structure is similar to the AO, except for the opposite sign over the Atlantic and the Pacific sectors. The score time series of EOF-2 indicates interannual variability with a 2–3-yr period. The apparent decadal variability seen for EOF-1 is not obvious for EOF-2.

Figure 7 illustrates the result of the same EOF analysis, but with the winter mean (DJF) data from 1950 to 1999. The structure of the AO in Fig. 7 is nearly identical to that in Fig. 5. The variance increases from 5.7\% for the daily data to 13.9\% for the monthly mean data and to 21.0\% for the winter mean (DJF) data. EOF-2 explains 10.3\% of the total variance of the winter mean data. The structure indicates an NAO-like pattern over the Atlantic sector while the Pacific sector clearly indicates a PNA-like pattern, as discussed by Wallace and Thompson (2002).

The time spectrum of the daily AO index for EOF-1 is plotted in Fig. 8. The result is characterized by a red noise spectrum over the higher frequency range. The red noise turns to white noise for the period longer than 4 months. It is interesting to note that the AO is the most dominant mode from the daily timescale to the decadal timescale, but there is no obvious peak in the time spectrum.

With the same procedure as to obtain the AO in Fig. 5 from the state variable \(w_t\), we can examine the most dominant mode for the external forcing \(s_t\). Figure 9 illustrates the structure of the EOF-1 for the external forcing computed with the daily data. The structure clearly shows the characteristics of synoptic-scale disturbances of zonal wavenumber 6 in the midlatitudes, located along the Pacific storm track. EOF-2 has a similar structure as EOF-1, but with a phase shift by 90\° for the zonal wavenumber 6. The combination of these two modes represents the progressive zonal wavenumber 6 along the Pacific storm track. A similar synoptic-scale forcing is seen along the Atlantic storm track in the higher EOF modes. These forcings represent the barotropic–baroclinic interactions \(g_t\) in (7) associated with baroclinic instability as parameterized by Tanaka (1998). The storm track analyzed by the transient bar-
Fig. 9. First two EOFs components of the external forcing of the barotropic component of the atmosphere, analyzed by the forcing $\xi$, with the daily NCEP-NCAR reanalysis from 1950 to 1999.
Barotropic Energy (S-Model)

Perpetual January Run

![Graph showing barotropic energy over time](image)

Fig. 10. Daily time series of barotropic energy for the perpetual Jan run of the barotropic S mode for 51 yr. The upper and lower lines represent zonal \((n = 0)\) and eddy \((n \neq 0)\) energy, respectively. (Unit: \(10^9 \text{ J m}^{-2}\)).

Barotropic height (not shown) is located rather north and downstream of the jet exit region as a response to the baroclinic forcing.

It is important to note that the major energy source of the barotropic component of the atmosphere appears in the synoptic-scale baroclinic eddies. The major response to the forcing, however, appears as the zonally symmetric annular mode. The nonlinear response to the synoptic-scale forcing may be connected by the upscale energy cascade governed by the internal barotropic dynamics.

4. Dominant modes of the model atmosphere

a. Perpetual January run of the barotropic S model

In this section, the barotropic S model of (7) is integrated for 51 yr starting from the initial condition of 0000 UTC 1 January 1950 under the perpetual January condition. The time series of zonal energy \((n = 0)\) and eddy energy \((n \neq 0)\) are plotted in Fig. 10. Since the initial condition is set on 1 January 1950, the abscissa is numbered, for convenience, from 1950 to 2000. The mean energy levels are approximately 12 and \(4 \times 10^9 \text{ J m}^{-2}\) for zonal and eddy energies, respectively. Those energy levels are consistent with winter observations in Fig. 4. The energy levels fluctuate with respect to time within 10 to \(14 \times 10^9 \text{ J m}^{-2}\) for zonal energy and 3 to \(6 \times 10^9 \text{ J m}^{-2}\) for eddy energy, respectively. No smoothing is applied in this figure. The fluctuation may be regarded totally as the natural variability of the model atmosphere since all model parameters and forcing parameters are fixed.

b. EOF analysis of the model atmosphere

Figure 11 plots the mean barotropic height for the perpetual January run for 51 yr. There are three major troughs over Japan, the east coast of the United States, and Europe. Although the details are different, to some extent, the overall features of the winter climate in Fig. 1 is simulated. The locations of storm tracks analyzed for the model atmosphere (not shown) agree quite well with observations. Therefore, both mean and transient fields are well described by the barotropic S model.

The EOF analysis is conducted for the time series of the model atmosphere with the same method used for the observed atmosphere discussed in section 3. Figure 12 illustrates the first two EOF components of the model atmosphere analyzed by the time series of \(w_1\). For the 50-yr model run with the sample size of \(N = 74,512\), the eigenvalues are well separated according to the significance test by North et al. (1982). The structure of EOF-1 indicates a negative region in high latitudes surrounded by a positive region in the midlatitudes. The positive region shows two peaks over the Pacific and Atlantic sectors. The zero crossing is located around 60°N. The structure is almost identical with the AO analyzed by the observed atmosphere in Fig. 5. Hence, EOF-1 in the model atmosphere may be identified as the AO. The positive peak over the Pacific sector is elongated in the zonal direction, which disagrees to some extent with observation. The elongated positive area in the observed Atlantic sector is now disconnected.
Barotropic Height

Perpetual January Run

Fig. 11. Mean barotropic height (in m) for the perpetual Jan run for 51 yr.

at the eastern Atlantic in the model. Although there are some differences, the overall features agree quite well with the observation in Fig. 5.

The score time series (365-day running mean) of the EOFs are plotted in Fig. 13. The time series of EOF-1 show a large interannual variability with 2–3-yr periods, similar to the observation in Fig. 6. The decadal variability seems to be absent for the model. EOF-1 explains 15.6% of the total variance of the daily data. The large contribution of the AO may be a result from the relatively weak transient eddies in the synoptic and short waves in the model atmosphere. It is evident from the result that the natural variability associated with the AO can be produced by the internal dynamics of the barotropic atmosphere.

EOF-2 explains 7.9% of the total variance of the daily data. There is a positive peak over the Atlantic sector that extends over North America. The Arctic is negative and the Pacific is also negative. The structure of EOF-2 agrees well with the observed EOF-2 in Fig. 5. The main discrepancy may be seen in the location and strength of the positive peak over Russia. Yet, the overall agreement with observation suggests that the barotropic S model captures not only the essential features of EOF-1 but also that of EOF-2 of the observed atmosphere. The score time series of EOF-2 indicates a larger interannual variability than that of the observation. The dominant periodicity of 2–3 yr agrees with the observation in Fig. 6.

We have compared the structures and the score time series of EOF-3 and EOF-4 with those of the observation (not shown). The result shows not only EOF-1 but also EOF-2 to EOF-4 agree fairly well with the observed EOFs. Therefore, it is confirmed in this study that the major characteristics of the atmospheric low-frequency variability have been captured satisfactorily by the barotropic S model.

The time spectrum of the daily AO index for the model atmosphere is plotted in Fig. 14. The red noise spectrum at the high-frequency range extends up to
Fig. 12. As in Fig. 5, but for the perpetual Jan run of the barotropic S model for 51 yr.

Barotropic Component of Geopotential Height

EOF-1 (15.6%)
the period of 4 months, then it shifts to the white noise spectrum at the low-frequency range. There is a minor spectral peak at 2-yr period, and the decadal variability is absent in the result. According to the result of the 500-yr perpetual January run (not shown here), we confirmed that the white noise extends all the way to the low-frequency range. In the observed atmosphere in Fig. 8, the shift from red noise to white noise occurs gradually around the same period. The spectrum is similar to that argued by James and James (1992). Since the white noise is characterized by the variability that has no memory in the past, the clear shift from the red noise to white noise suggests that the barotropic model has a memory up to 4 months as long as the AO index is concerned. The result suggests that the AO index has no specific periodicity, and the variation may be considered as a natural variability of stochastic random noise.
5. Energetics of the model atmosphere

a. Energy spectrum and conversions

In this section, the energy spectrum and conversions of the model atmosphere is compared with observations. Figure 15 plots the barotropic \((m = 0)\) energy spectrum of \(E_r = E_{\text{eddy}}\) as a function of the scale index \(|c_1| = |c_{\text{eddy}}|\) evaluated for the model atmosphere for 1 yr. In order to avoid the influence of the initial condition, the second year of the time integration was analyzed. The energy distribution compares well with the observations in Fig. 2. The peak of the energy spectrum for the zonal eddy \(n \neq 0\) is determined by Rhines’ scale, where the inverse energy cascade terminates (Rhines 1979). The spectrum for \(n = 0\) coincides with that of \(n \neq 0\) for a smaller \(|c_1|\), but it continues to increase for larger \(|c_1|\). The inverse energy cascade continues to an even larger scale for \(n = 0\). The energy level is lower than observations for short waves with smaller \(|c_1|\).

The AO or NAM can be characterized by variations in zonally symmetric component of \(n = 0\). It is an interesting subject to analyze the energy source and sink for the zonal flows in the model atmosphere. The mean (1 yr) energy levels and energy conversions are computed based on Tanaka (1998), and the result is listed in Table 1 as functions of zonal wavenumber \(n\). Both total energy (EN) and total external forcing (FN) can be assessed directly from the observation. The contributions from the climate forcing \(f_\text{c}\) are added, for convenience, to the system forcing (AB).

The mean zonal energy is \(12.1 \times 10^3 \text{ J m}^{-2}\), which is comparable with observations. Two spectral peaks are seen for EN at the wavenumbers \(n = 1\) and \(3\), and the energy level decreases for short waves. The energy levels for short waves are smaller than the observation in Fig. 3 since there is no energy source in this range. The synoptic energy source is transformed mostly to large-scale motions and little comes to short waves under the restriction of the barotropic dynamics.

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TABLE 1. Time-mean (1 yr) energy \((10^4 \text{ J m}^{-2})\) and energy conversions \((10^{10} \text{ W m}^{-2})\) for the barotropic S model at functions of the zonal wavenumber \(n\). The symbol EN represents total energy, AB system forcing, BC baroclinic instability, NL nonlinear wave–wave interaction, DS surface friction, DF diffusion, and FN total external forcing.
Energy Spectrum
Barotropic S-Model

\[ \Pi_{n,m} \quad m = 0 \]

\[ \begin{align*}
\text{Total Energy (J m}^{-2}) & \\
10^7 & \\
10^6 & \\
10^5 & \\
10^4 & \\
10^3 & \\
10^2 & \\
10^1 & \\
10^0 & \\
10^{-1} & \\
10^{-2} & \\
10^{-3} & \\
10^{-4} & \\
\end{align*} \]

\[ \text{Phase speed} \]

\[ \begin{align*}
\text{n = 0} & \quad \text{Rotational mode} \\
\text{n = 1-20} & \quad \text{Rossby mode} \\
\end{align*} \]

Fig. 15. As in Fig. 2, but for the perpetual Jan run of the barotropic S model for 1 yr. The black and white dots denote \( n = 0 \) and \( n \neq 0 \), respectively. (Unit: J m\(^{-2}\).)

sonable compared with observations (Tanaka and Kung 1988; Tanaka 1998).

The result of the energetics analysis indicates two major energy sources at synoptic-scale eddies and planetary waves. The energy is redistributed mostly to zonal motion by the nonlinear upscale energy cascade within a framework of the barotropic dynamics (see Fig. 2). The nonlinear upscale energy cascade by NL is the sole energy supply for the zonal motions where the NAM is contained.

b. Energetics of the anomaly field

The AO or NAM can be analyzed as the variation in the anomaly field, especially for the zonally symmetric component of \( n = 0 \). For this reason, a similar energetics analysis is conducted for the anomaly field of the model atmosphere in this section. The energy spectrum is evaluated for anomaly \( w'_a \) and is plotted in Fig. 16 as a function of \( |c| \). We can notice that the zonal energy (black dots) is reduced substantially compared with that in Fig. 15. The result suggests that the large fraction of zonal energy is contained in steady motions. The spectral peak in the zonal eddy (white dots) is also reduced, indicating that a large fraction of planetary waves are explained by the steady component induced by topographic forcing. However, the energy levels for the synoptic and short waves are comparable with those in Fig. 15, showing the dominant transient motions for these eddies. The energy peak for the anomaly field appears in the intermediate scale of zonal field. The energy peak level for \( n = 0 \) is as large as that of zonal eddy \( n \neq 0 \).

It is in this range where the characteristics of the AO or NAM are contained.

The mean (1 yr) energy levels and energy conversions for the anomaly field are listed in Table 2 as functions of
zonal wavenumber \( n \). All variables including the nonlinear term are separated in the mean and anomaly, and the energy budget is computed with those anomalies. The EN levels are comparable for \( n = 0, \ldots, 5 \), and the peak is seen at \( n = 1 \). The BCs are the main energy source of the anomaly for the synoptic-scale transients. The NL term acts to reduce the anomaly for synoptic eddies. On the contrary, the nonlinear term appears to be the major energy source of the anomaly in planetary waves and the zonal motions. Here DS and DF act to reduce the anomaly for all scale of motions. It is interesting to note that AB is not the energy source but a sink of the anomaly field in planetary waves. The system forcing represents mostly the topographic forcing. The large energy source of AB in Table 1 is thus explained by a steady forcing induced by the topography. This term rather acts to reduce the anomaly in planetary waves.

A large time variation is a notable feature of the NL term. Therefore, a large fraction of the time variation associated with NAM can be explained by the chaotic fluctuation of NL, which is the sole energy source of the transient anomalies in planetary waves and zonal motions.

6. Discussion and conclusions

In this study, a numerical simulation of the Arctic Oscillation (AO) is conducted using a simple barotropic model that considers the barotropic-baroclinic interactions as the external forcing. The model is referred to as a barotropic S model since the external forcing is obtained statistically from the long-term historical data solving an inverse problem. We have integrated the S model for 51 yr under the perpetual January condition.
and analyzed the dominant EOF modes in the model. The model results are compared with the EOF analysis of the barotropic component of the real atmosphere based on the daily NCEP–NCAR reanalysis for 50 yr from 1950 to 1999.

According to the result, the first EOF of the model atmosphere appears to be the AO similar to the observation. The annular structure of the AO and two centers of action at Pacific and Atlantic sectors are simulated nicely by the barotropic S model. The result suggests that the AO may be understood as the natural variability of the barotropic component of the atmosphere induced by the inherent barotropic dynamics, which is forced by the barotropic–baroclinic interactions. The EOF analysis of the model atmosphere shows not only that EOF-1 but also EOF-2 to EOF-4 agree fairly well with the observed EOFs. Therefore, the characteristics of the atmospheric low-frequency variability have been captured satisfactorily by the barotropic S model.

The EOF analysis is further applied for the external forcing of the barotropic component of the observed atmosphere. According to the result, the most dominant mode of the forcing is the barotropic–baroclinic interactions associated with baroclinic eddies of zonal wavenumber 6 located along the Pacific storm track. A similar synoptic-scale forcing is seen along the Atlantic storm track in the higher EOF modes. The energy source at the synoptic-scale eddies is transformed to zonally symmetric motions by the nonlinear upscale energy cascade, which is the sole energy source of the zonal wavenumber $n = 0$. Hence, the annular structure of the AO or NAM has been excited by the nonlinear upscale energy cascade from active transient eddies along the storm tracks. The positive feedback between transient eddies and zonal motions can be one of the possible excitation mechanism of the AO (Lorenz and Hartmann 2001; Takaoka and Tokinaga 2002).

There are two active storm tracks over the Pacific and Atlantic sectors. These two storm tracks are known to behave independently, indicating insignificant correlation with each other (Ambaum et al. 2001). When the Atlantic storm track produces positive and negative height anomalies over the Atlantic and the Arctic, respectively, the annular structure appears excited by the energy supply to the zonal motion. Similarly, when the Pacific storm track produces positive and negative height anomalies over the Pacific and the Arctic, respectively, the same annular structure appears excited by the energy supply to the zonal motion. Although the activities of the two storm tracks are independent and not correlated with each other, the statistical EOF analysis reveals a single negative height anomaly over the Arctic and two positive height anomalies over the Pacific and Atlantic sectors, as demonstrated by Deser (2000), Ambaum et al. (2001), and Wallace and Thompson (2002). EOF-2 indicates opposite signs over the Pacific and Atlantic sectors in order to describe the independent behavior between the two sectors as demonstrated in this study with the barotropic S model.

The zonally symmetric annular mode would be excited when the storm track is distributed uniformly as seen in the SAM. In the Northern Hemisphere, the AO or NAM would be excited both by transient synoptic eddies along the storm tracks and standing planetary waves. The upscale energy cascade from those eddies or waves to zonal motions would produce the largest-scale annular mode of $n = 0$, which can exist in both hemispheres. The zonally symmetric annular mode can be excited by various arbitrary forcing such as a volcanic effect, global warming, or the Milankovitch cycle because the energy of the impact is transformed eventually to the zonal flow by the characteristics of barotropic dynamics. In conclusion, the AO can be dynamically understood as the internal mode of the barotropic atmosphere where the largest-scale annular mode is excited by the nonlinear upscale energy cascade from various energy sources.

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